

# Applications of Serpent's Fission Matrix Capability

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*Penn State University*

*Ken and Mary Alice Lindquist Dept. of Nuclear Engineering*

*Presented at the Serpent User's Group Meeting*

*Atlanta, GA*

*10/17/2019*



# The Fission Matrix method is in theory fairly simple

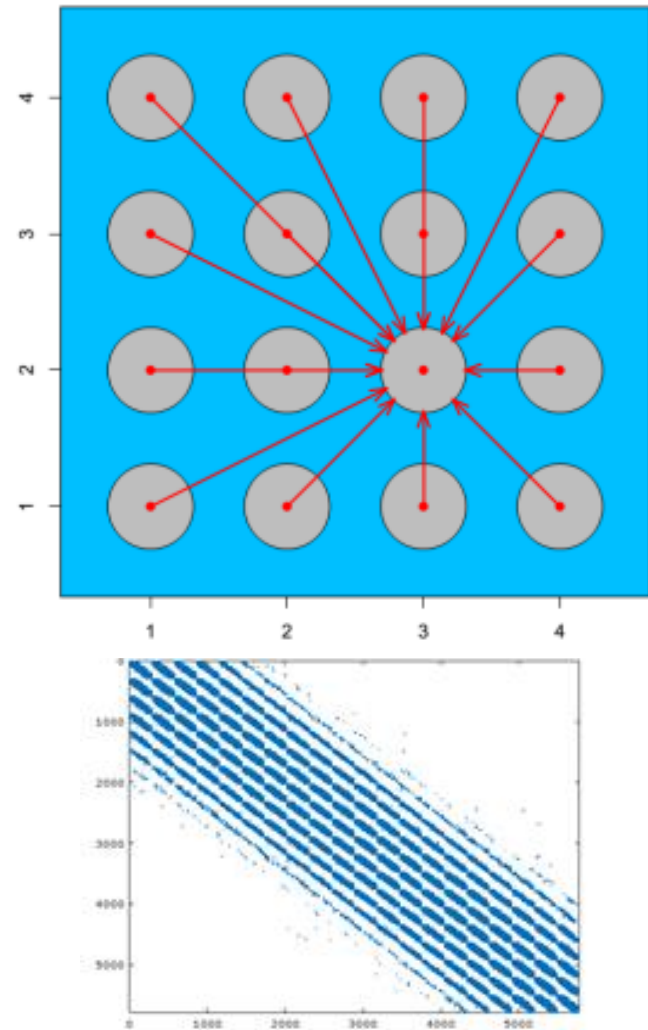
$$F_i = \frac{1}{k} \sum_{j=1}^N a_{i,j} F_j$$

$F_i$  = fission neutrons in cell  $i$

$a_{i,j}$  = number of neutrons created in cell  $i$   
due to a neutron born in cell  $j$

$$\vec{F} = \frac{1}{k} A \vec{F}$$

- Calculate  $a_{i,j}$  using Monte Carlo
- Solve linear system for  $\vec{F}$ ,  $k$



# Fission matrices have been used for several purposes

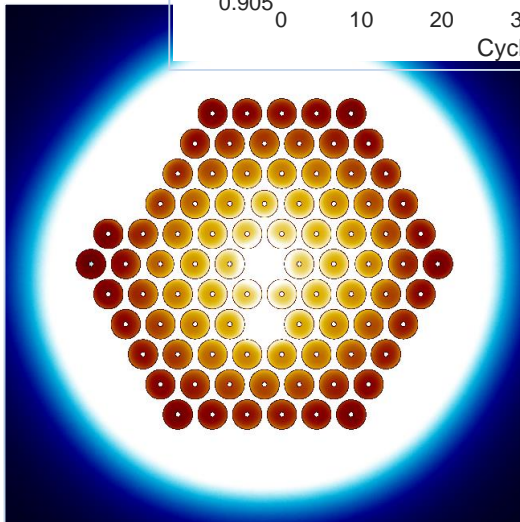
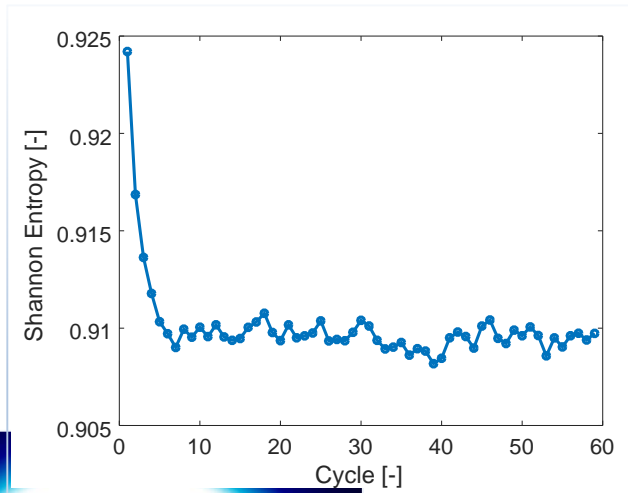
- Criticality convergence acceleration / checking
- Higher-order modes / importance
- Two-step hybrid calculation
  - Pre-calculate fission matrices for “representative” problems
  - Use pre-calculated data to estimate fission matrix for particular problem

# Fission matrices have been used for several purposes

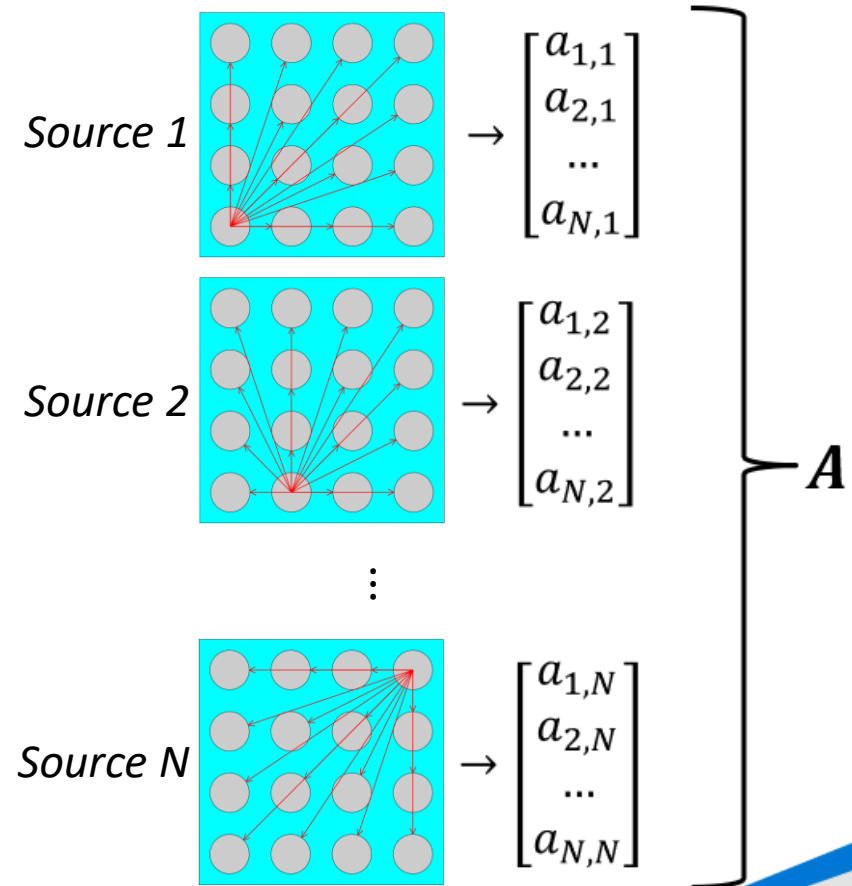
- Criticality convergence acceleration / checking
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- Two-step hybrid calculation
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# Serpent can tally fission matrices in a set of fixed-source or a criticality calculation

Criticality

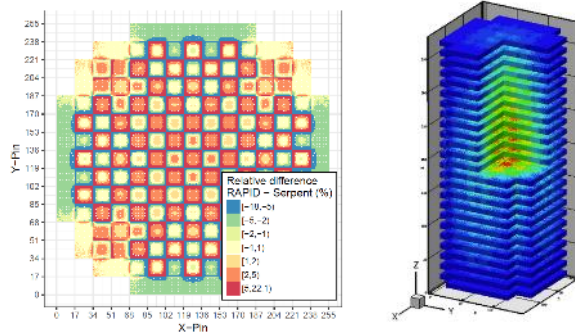


Fixed Source

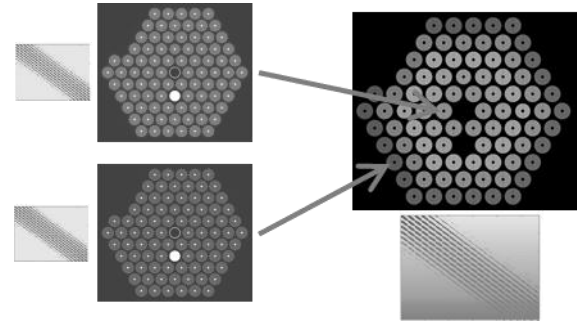


# Presentation Outline

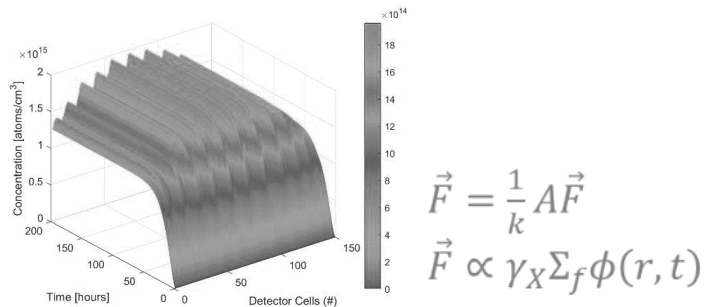
## RAPID Matl. Correction Ratios (Donghao He)



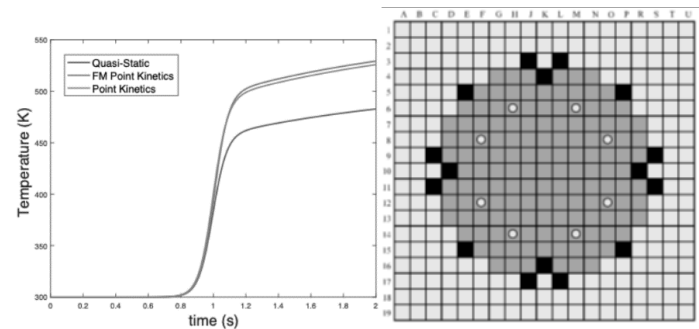
## Temperature/Control Methods for TRIGA and NTP (Adam Rau)



## Fission-Matrix-Based Xenon Transients (Jake Eichenlaub)

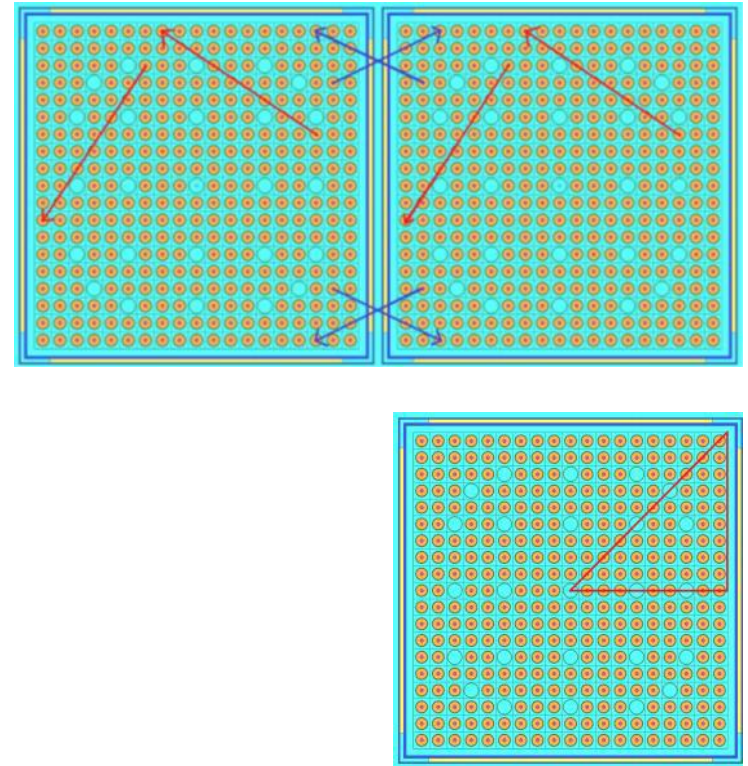


## Transient Fission Matrix in TREAT Reactor (Alvaro Pizarro-Vallejos)



# RAPID uses fission matrix methods for fast calculations

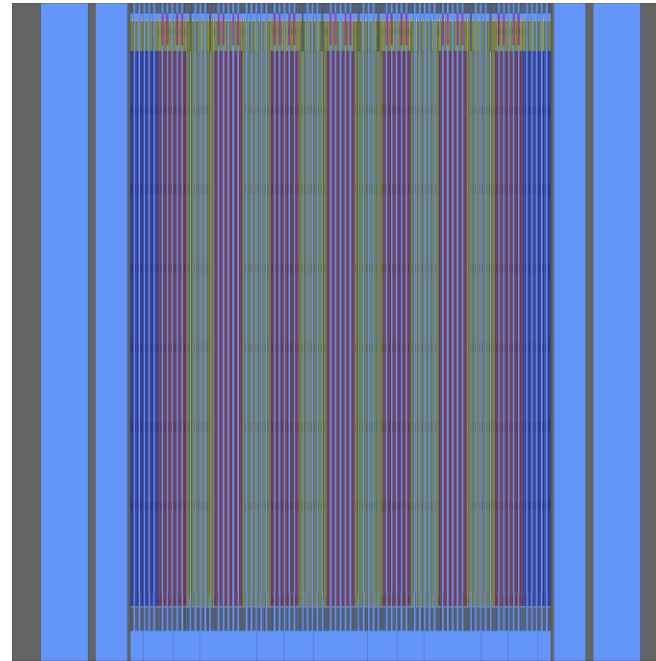
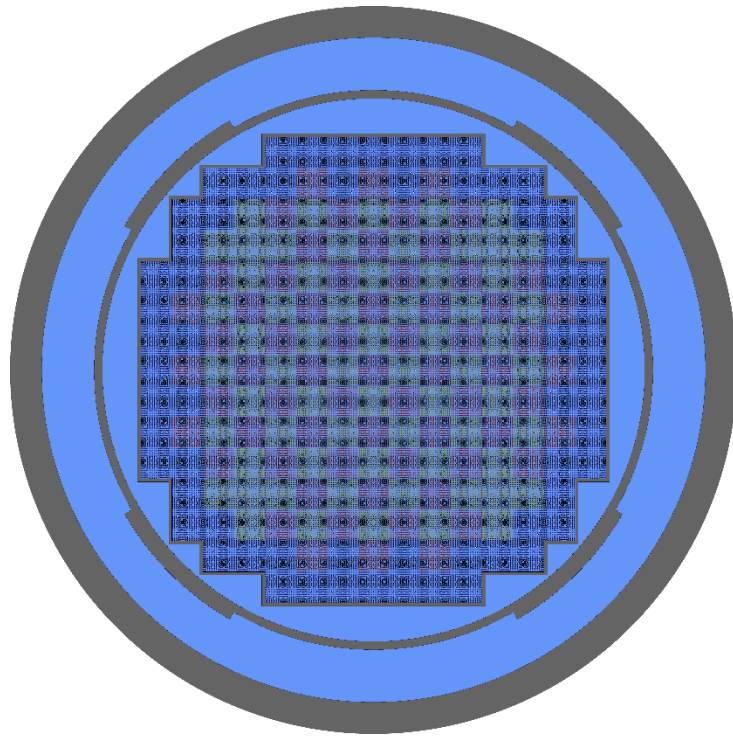
- Pre-calculate assembly-level fission matrices using *fixed-source* calculations
- Combine using geometric symmetry and similarity to simulate different loading patterns



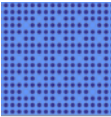
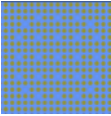
Walters et al., 2014



# RAPID has been applied to the BEAVRS benchmark



- Red: 1.6%
- Yellow: 2.4%
- Blue: 3.1%

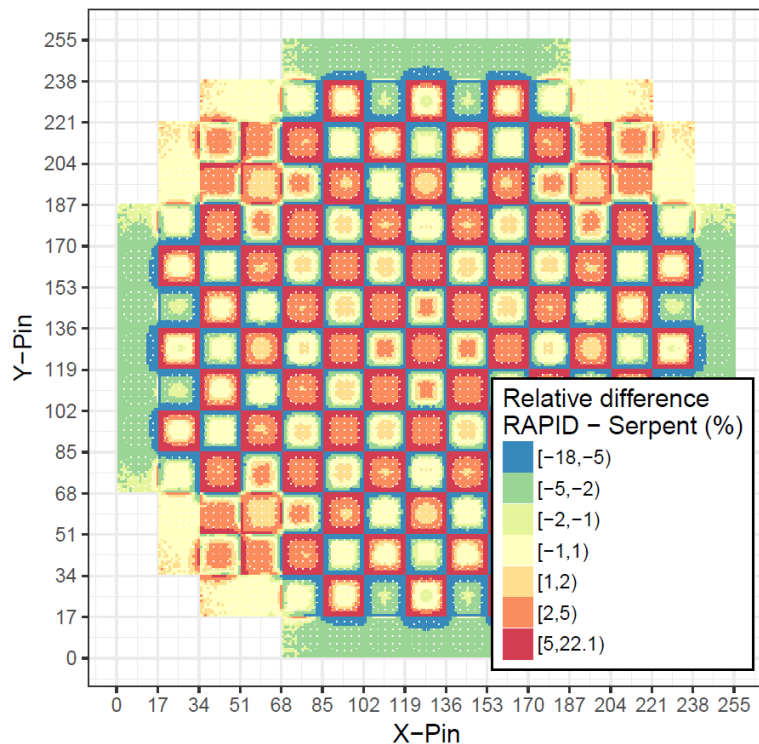


193 fuel assemblies

Each assembly has 264 fuel pins and 25 guide tubes in a  $17 \times 17$  lattice



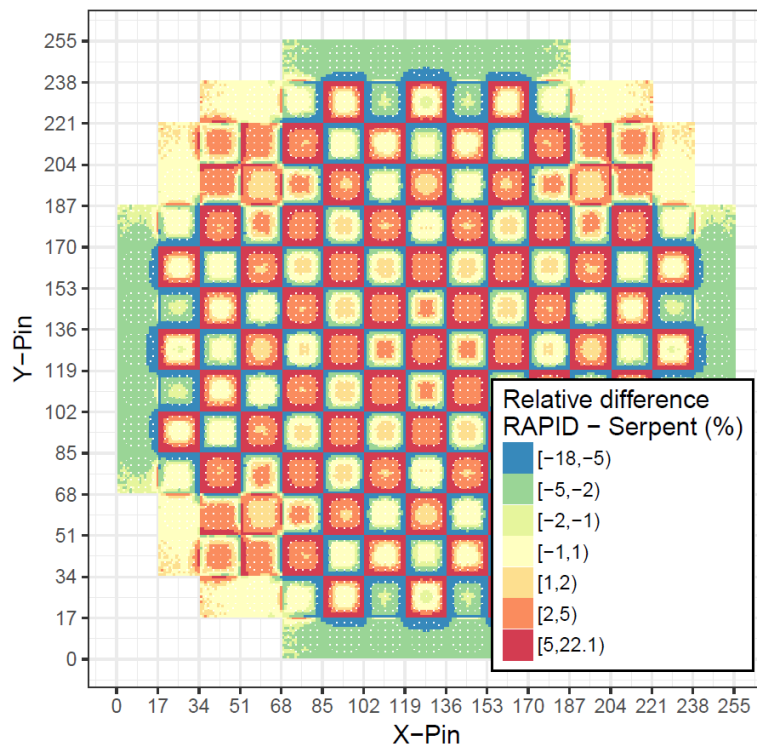
# Adjacent fuel assemblies with different enrichments may induce errors in RAPID



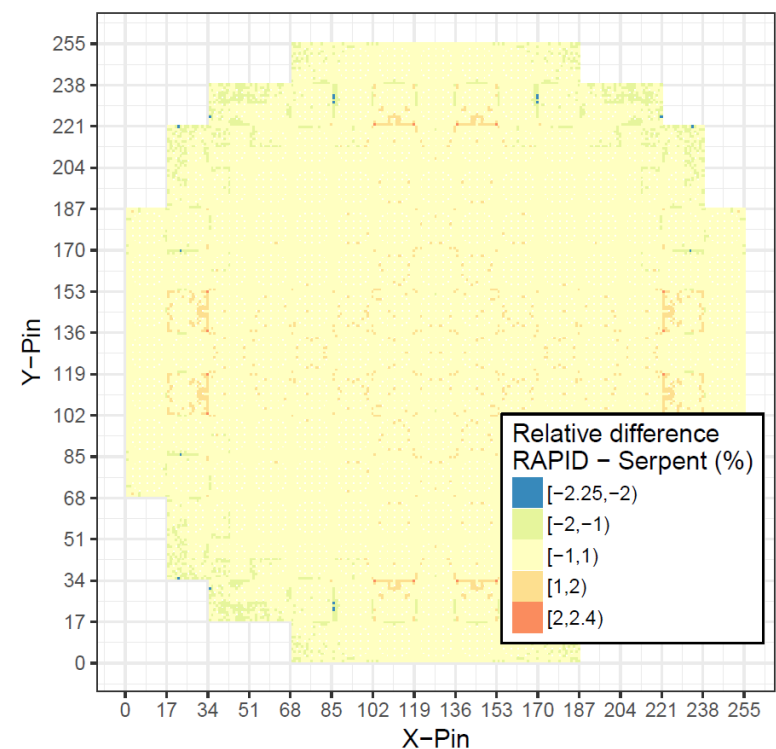
Code was originally designed for spent-fuel pools, where heterogeneity problems did not occur due to strong absorbers between fuel assemblies

# Methods were added to RAPID to correct enrichment-heterogeneity-induced errors

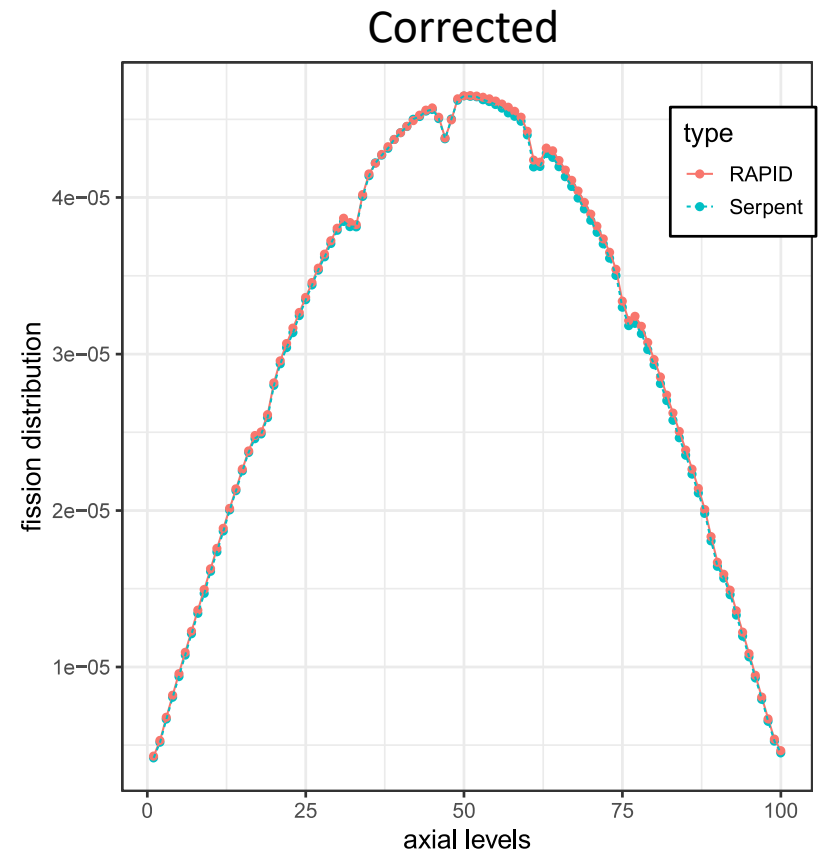
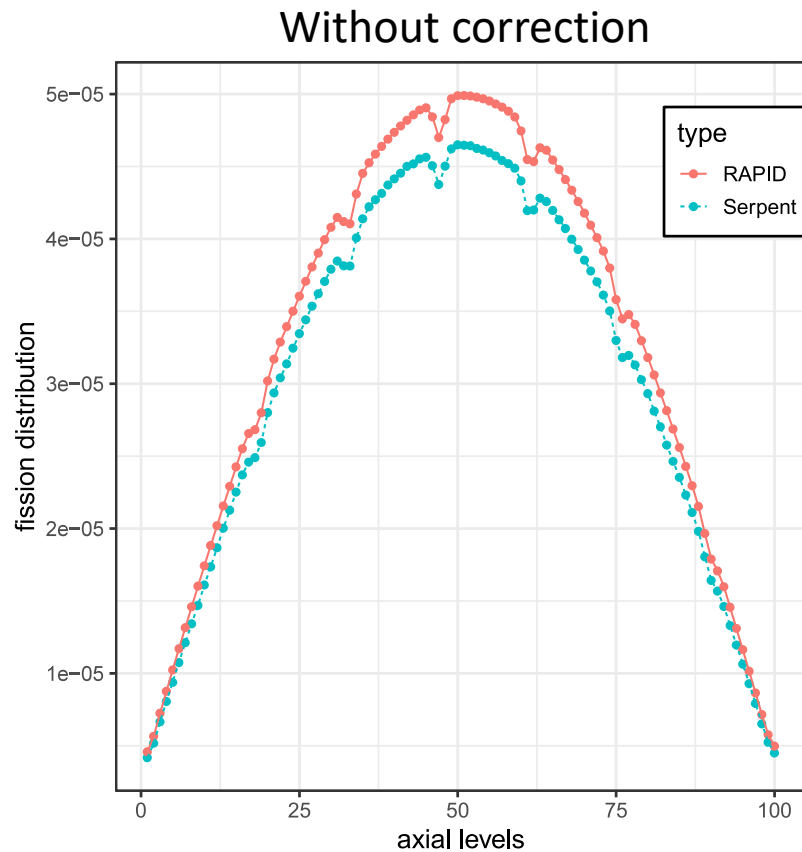
Without correction



Corrected



# Methods were added to RAPID to correct enrichment-heterogeneity-induced errors



# New methods incur only a small up-front computational cost

Calculation	Model Description		Number of Variations	Time (CPU-hr)	
	Geometry	Source		per Var.	Total
$\tilde{a}_{i,j}$ FM	1) Inf. assemblies	Single pin	$7 \times 39$	0.45	121.9
Refl. correction	2) Standard core	Criticality	1	3.8	156.2
	3) Inf. core	Fixed from (2)	1	50.5	
	4) Inf. radial core	Fixed from (2)	1	50.7	
	5) Inf. axial core	Fixed from (2)	1	51.2	
Enrich. correction	6) Four-assembly	Uniform	$3 \times 3$ (exact)	3.3	29.7
Total (exact $R$ )					337.5

New method

← Total

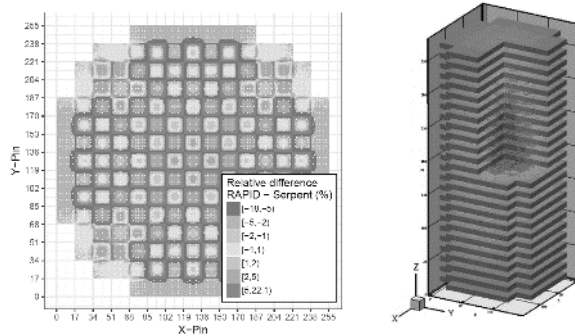
Correction method requires 9 fixed source calculations on four-assembly geometry

# Accuracy of $k_{eff}$ and 3D fission source estimation is improved

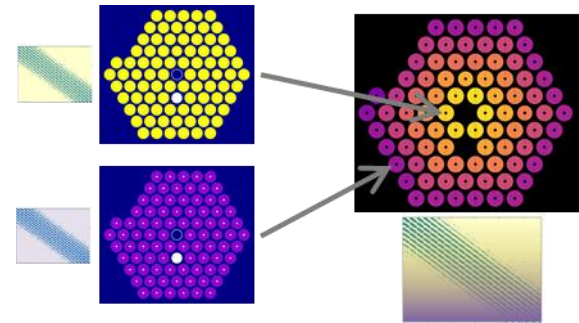
		Uncertainty (Serpent) and relative error (RAPID)					Wall-clock
		2D pin-wise			3D (100 axial levels)		Time
Method	$k_{eff}$	$k_{eff}$	RMS	MAX	RMS	MAX	(20 cores)
Serpent	1.00402	0.5 pcm	0.18%	0.43%	1.86%	11.35%	80 hr
RAPID (old)	1.00245	-156 pcm	6.26%	22.27%	6.63%	57.53%	2.4 min
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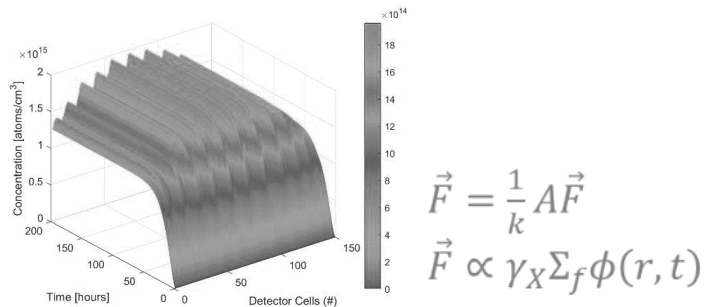
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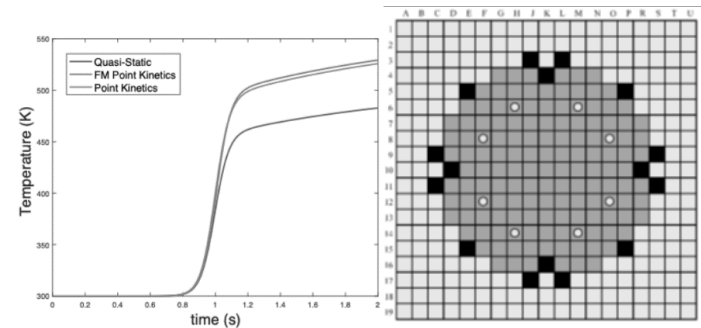
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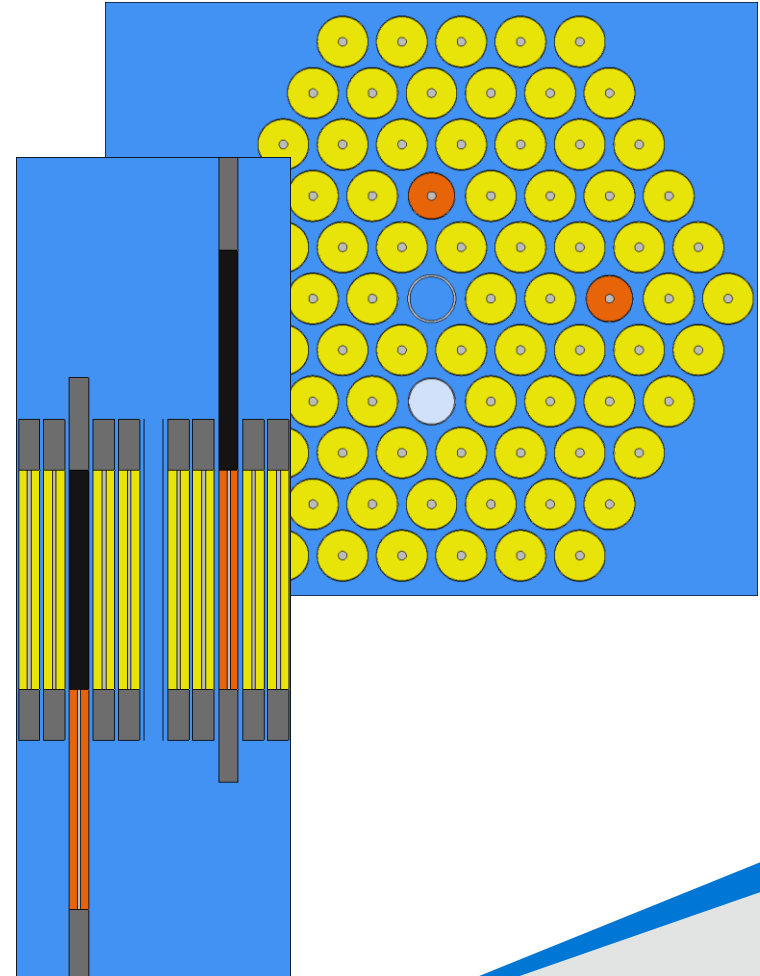


## Transient Fission Matrix in TREAT Reactor (Alvaro Pizarro-Vallejos)



# We have also developed fission matrix methods for the Penn State TRIGA reactor

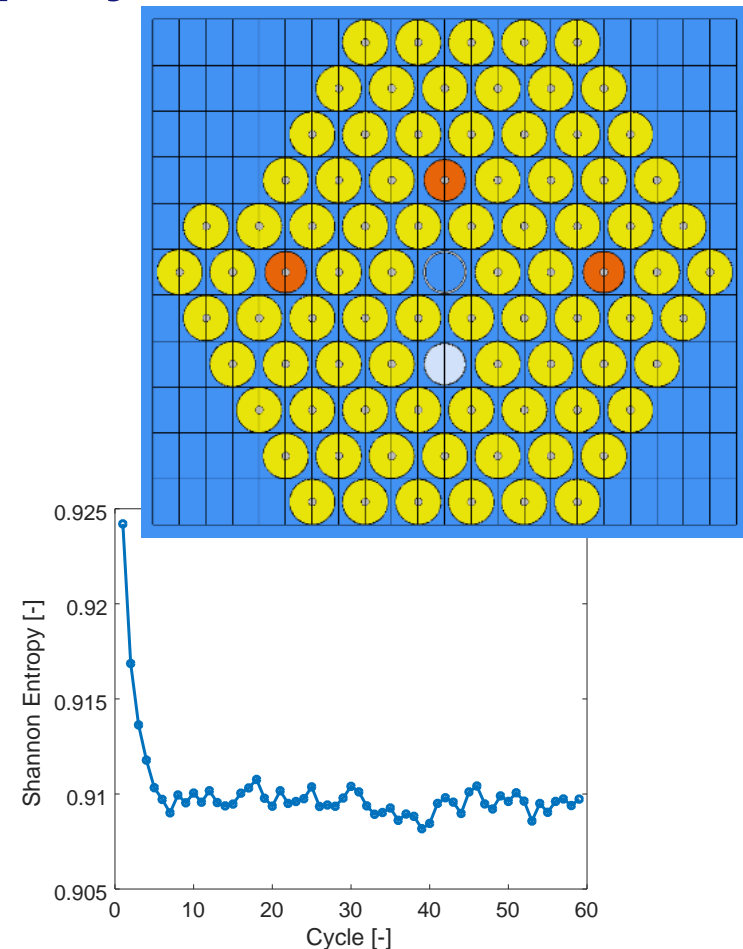
- TRIGA pool-type reactor
- $\text{UZrH}_x$  fuel (“cell effect” feedback)
- Four control rods
  - Three fuel-followed
  - One air-followed (pneumatically-actuated “transient” rod)
- Small core – 47 x 41 x 38 cm (approx.)
- Natural circulation cooling
- 1 MW nominal steady-state power





# Several Serpent options simplify this work

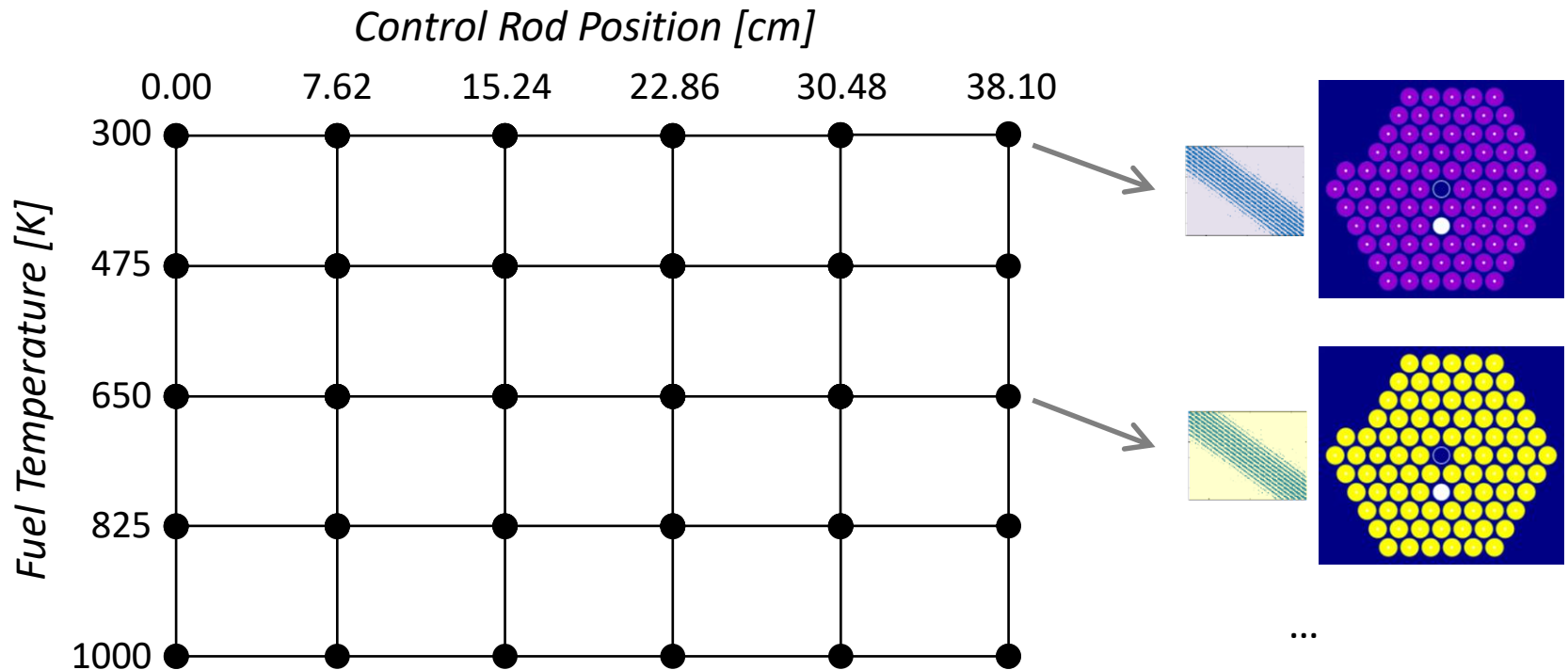
- “**set fmtx**” option for calculating fission matrix in criticality simulation
  - Easy to perform criticality calculations on small core
  - Cartesian grid compatible with regular hexagonal mesh
- Fuel temperature profiles simulated with **multiphysics interface**
  - Use same options even when temperature profile is uniform (since goal is code-to-code verification)



Set *fmtx*: <https://ttuki.vtt.fi/serpent/viewtopic.php?f=24&t=2098&p=5370&hilit=fmtx#p5370>

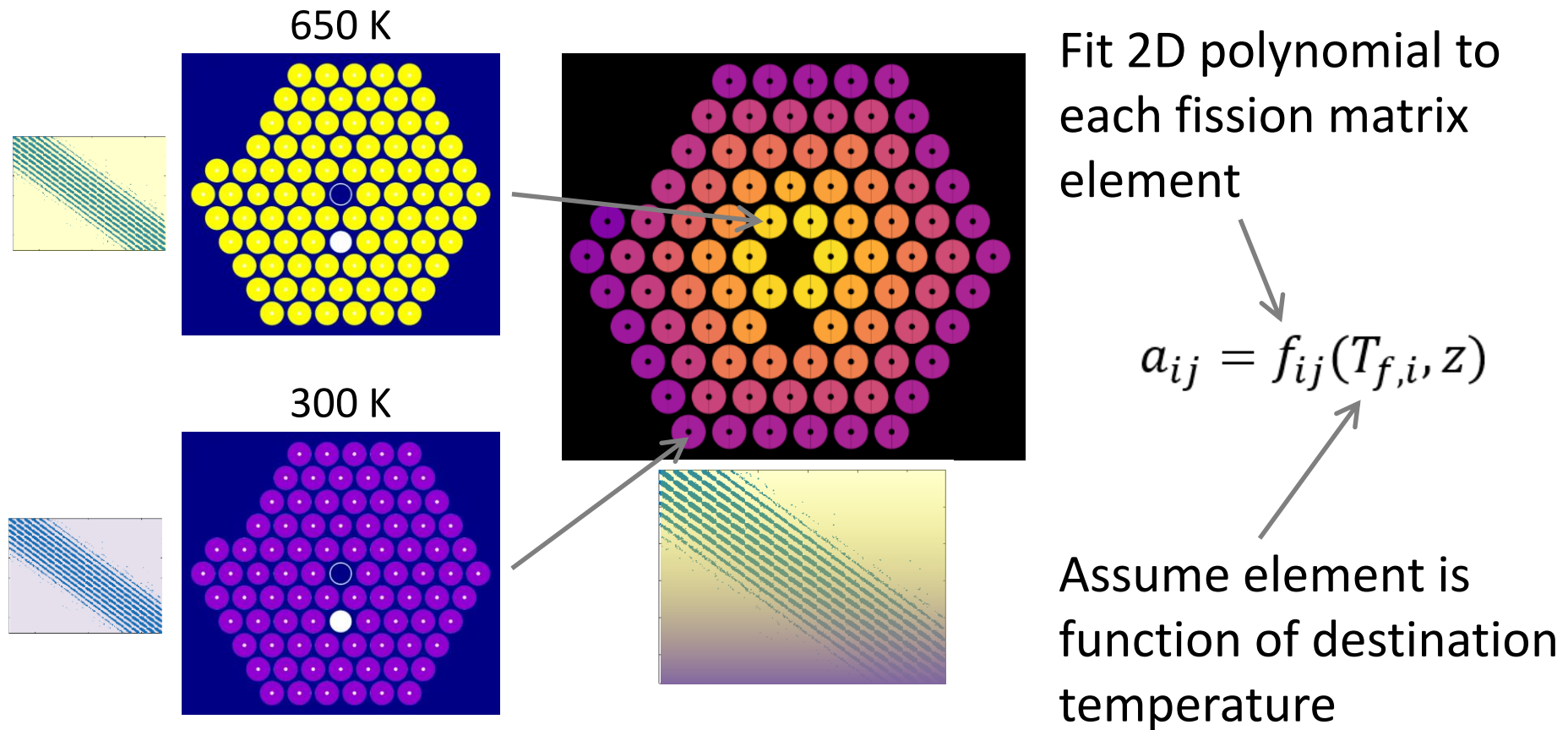
Multiphysics Interface: [http://serpent.vtt.fi/mediawiki/index.php/Multi-physics\\_interface](http://serpent.vtt.fi/mediawiki/index.php/Multi-physics_interface)

# We pre-calculate fission matrices at different uniform temperatures and control positions

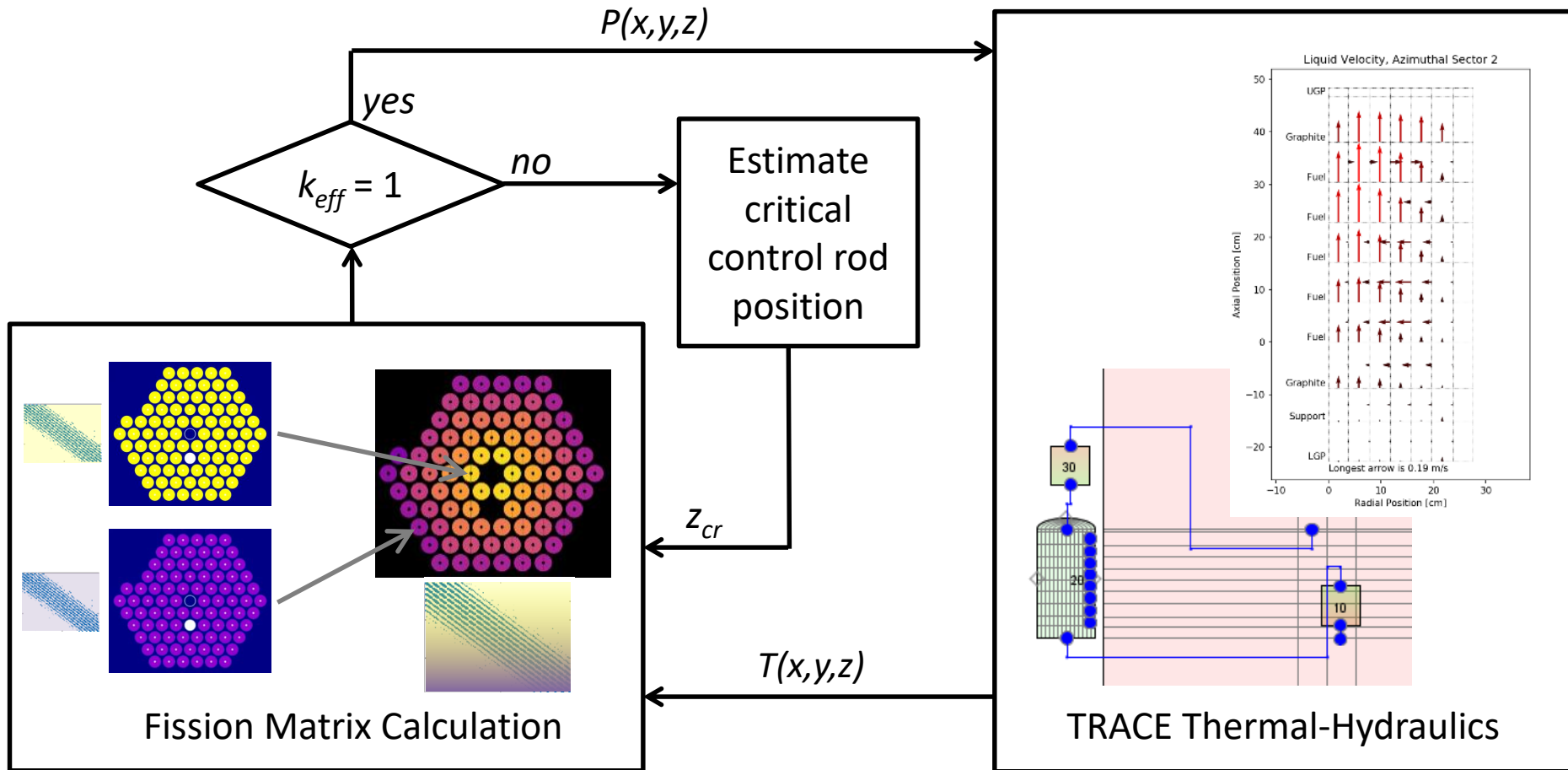


(5 uniform fuel temperatures) ·  
(6 control rod positions) =  
30 calculations

# Pre-calculated fission matrices can be combined to simulate any temperature profile

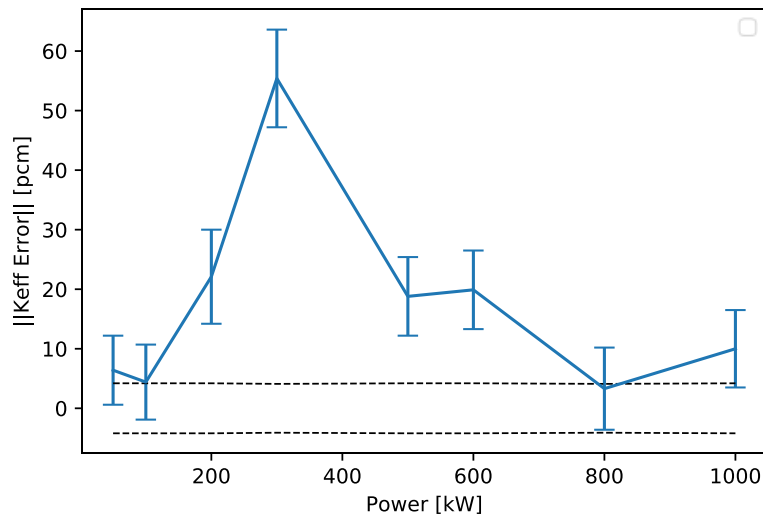


# Steady-state temperatures and rod positions were found by Picard iteration with TRACE

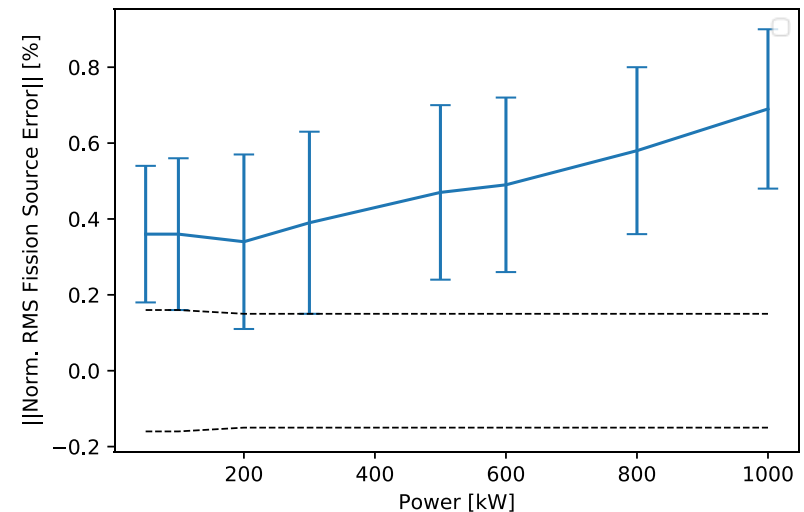


# Code-to-code verification is performed with Serpent

$k_{eff}$  Error



3D RMS Fission Source



- Critical temperatures / rod positions from 50 kW to 1 MW
- $k_{eff}$  within 60 pcm
- 3D RMS Source Error within 1%
- 3D Max. Source Error within 3%

# After pre-calculation, fission matrix solutions are completed in seconds

## For Database:

40,000 neutrons/cycle

12,000 active cycles

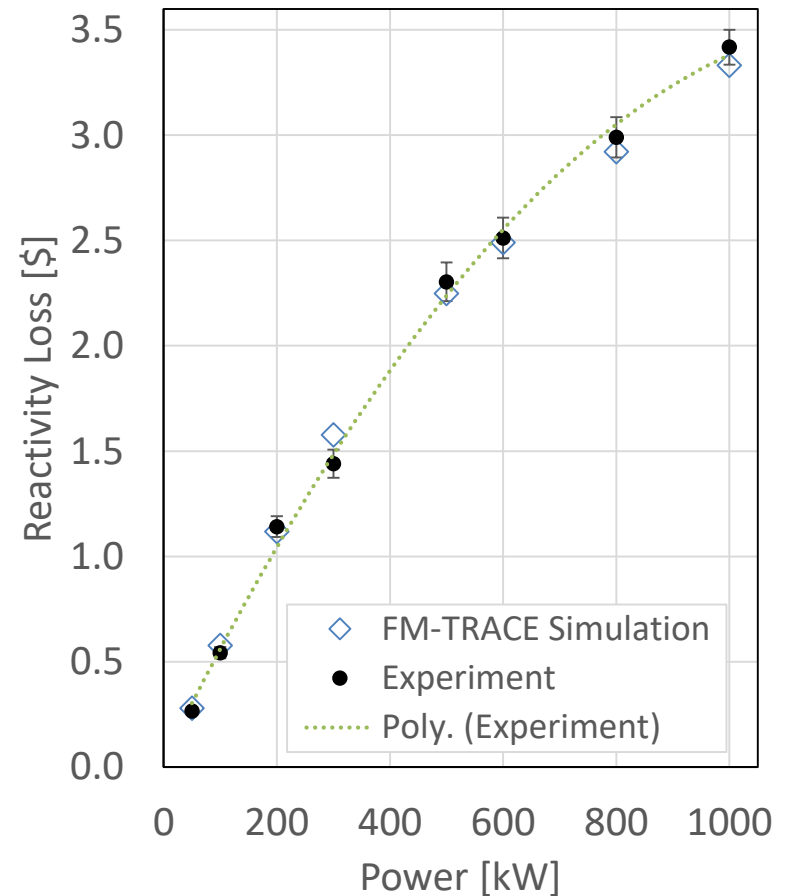
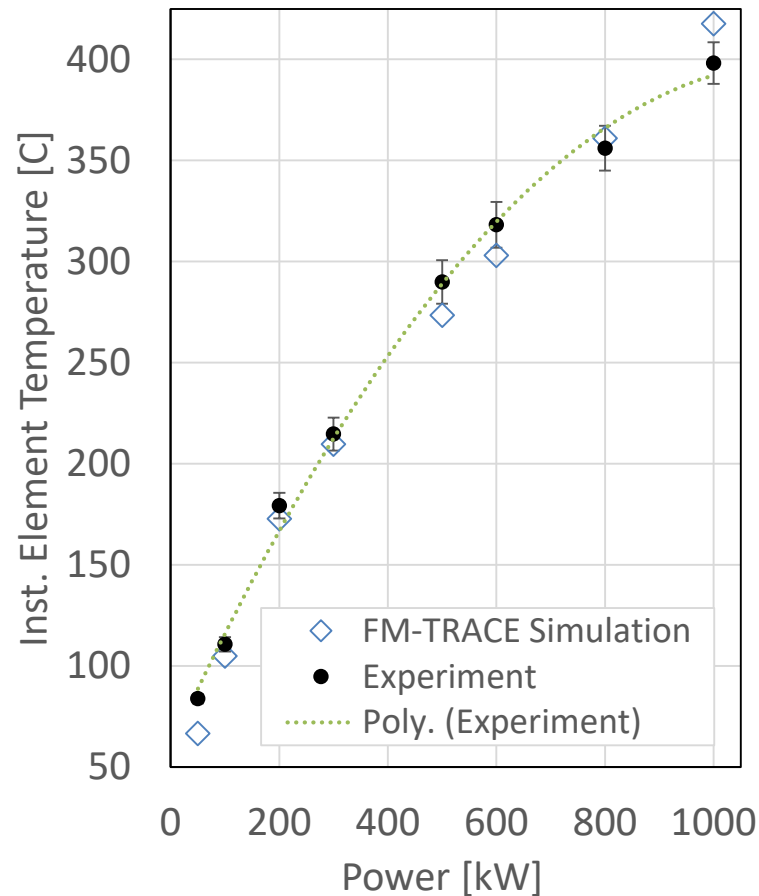
10 inactive cycles

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Fission Matrix Database Calculation (one calc.)	80 CPU-hrs
Fission Matrix Database Calculation (total)	2400 CPU-hrs
Fission Matrix Assembly / Solution (one calc.)	1.6 CPU-sec

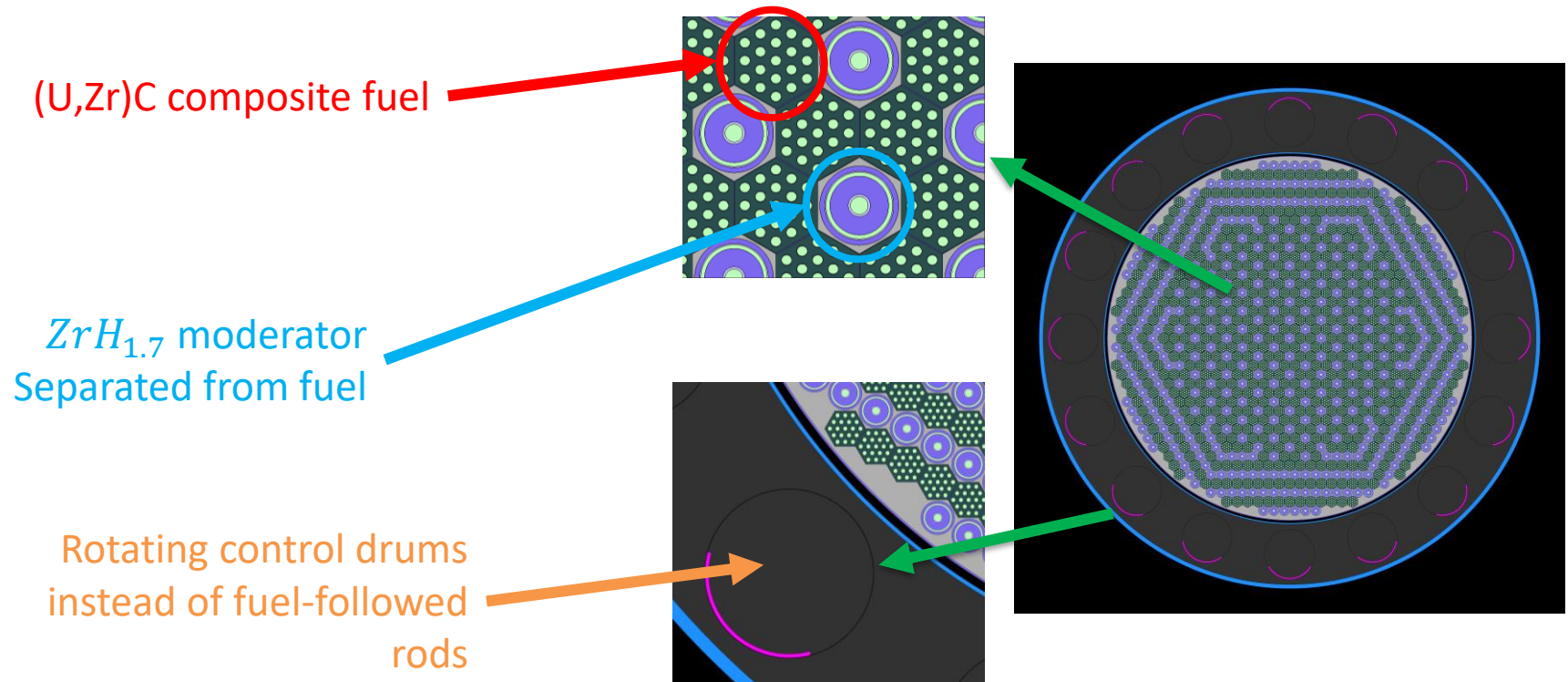
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# Method is validated against historic clean-core experimental data

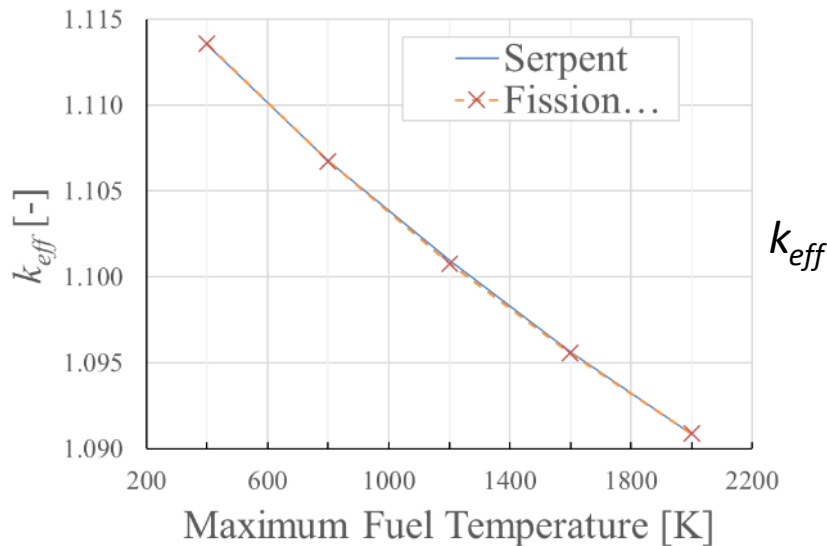




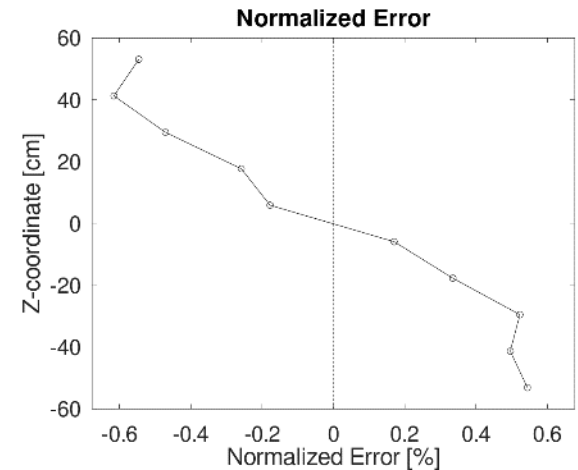
# The same methods have been applied to a Nuclear Thermal Propulsion type reactor



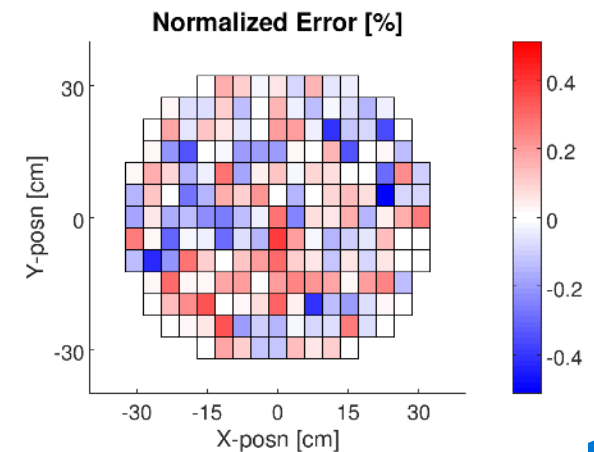
# Despite different application, method has similar accuracy on NTP and TRIGA



Axial Fission Source

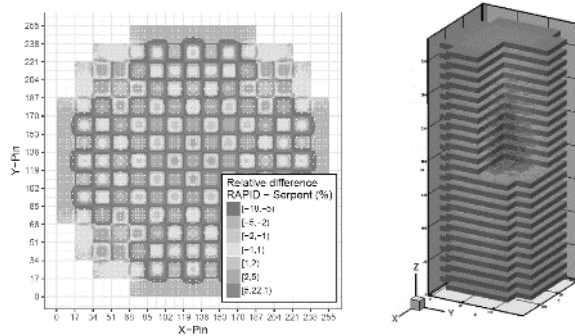


x-y Fission Source

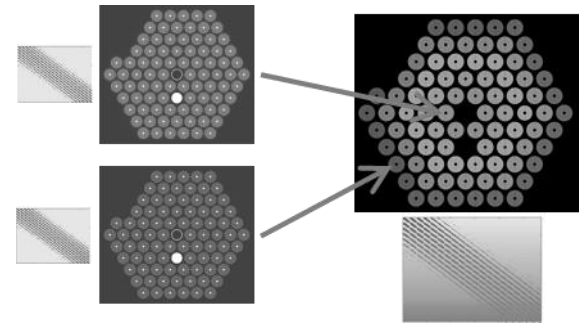


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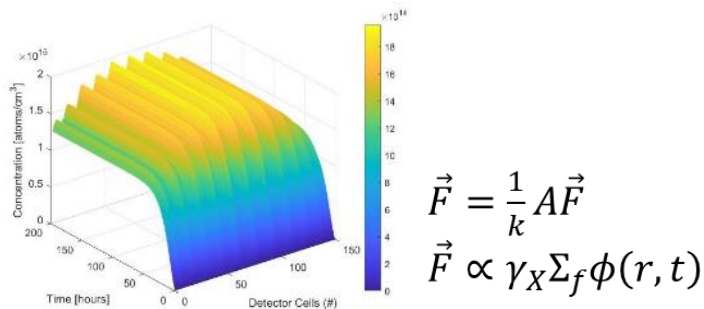
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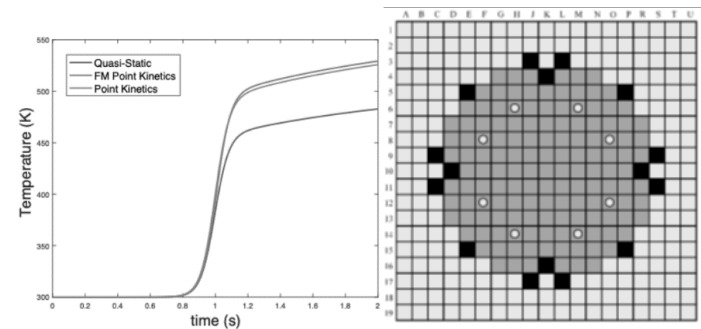
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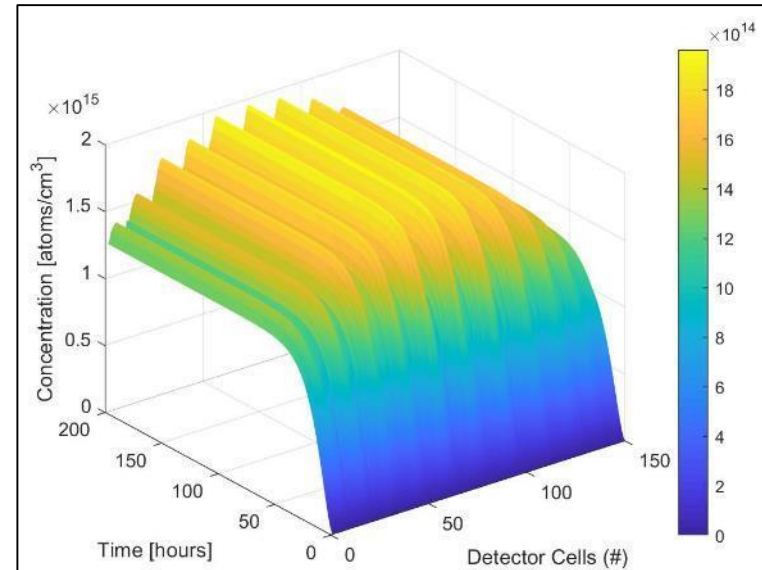
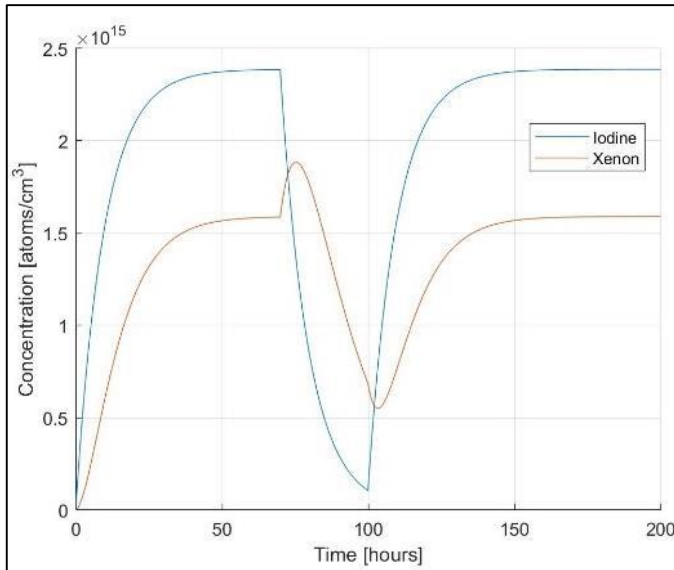
Fission-Matrix-Based Xenon  
Transients (Jake Eichenlaub)



Transient Fission Matrix in TREAT  
Reactor (Alvaro Pizarro-Vallejos)



# Short operation times require calculation of xenon dynamics



$$\frac{\partial I}{\partial t} = \gamma_I \Sigma_f \phi(r, t) - \lambda_I I(r, t)$$

$$\frac{\partial X}{\partial t} = \gamma_X \Sigma_f \phi(r, t) + \lambda_I I(r, t) - \lambda_X X(r, t) - \sigma_a^X \phi(r, t) X(r, t)$$

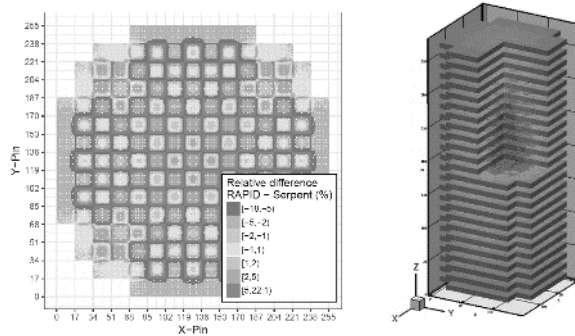
$$\vec{F} = \frac{1}{k} A \vec{F},$$

$$\vec{F} \propto \gamma_X \Sigma_f \phi(r, t)$$

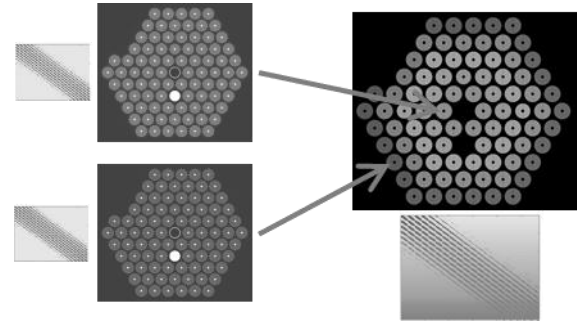
← Xenon production assumed proportional to fission source

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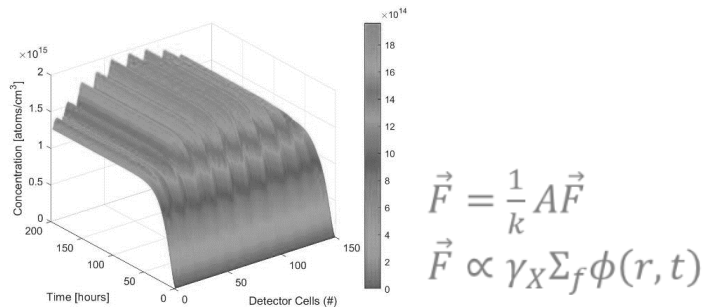
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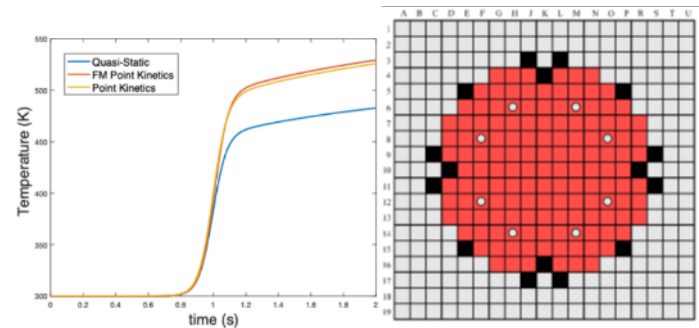
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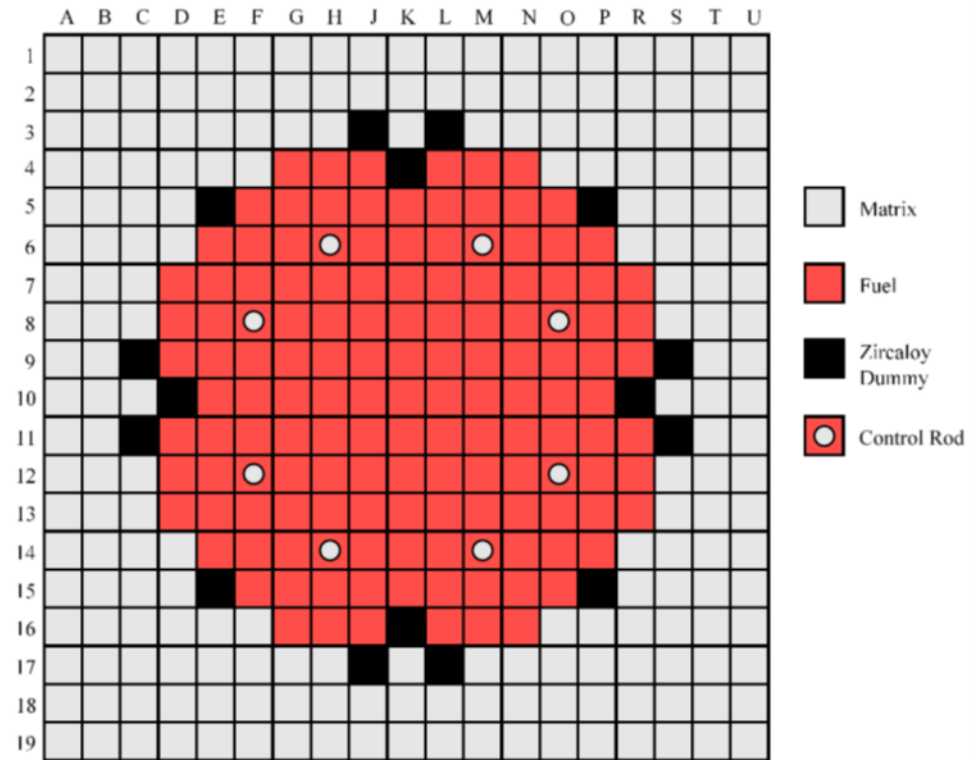
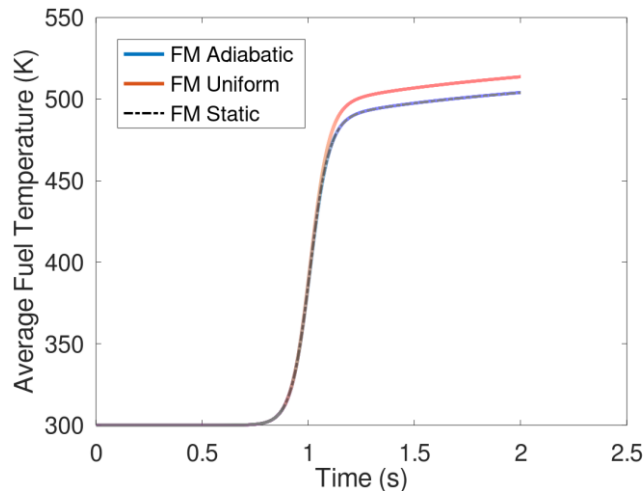
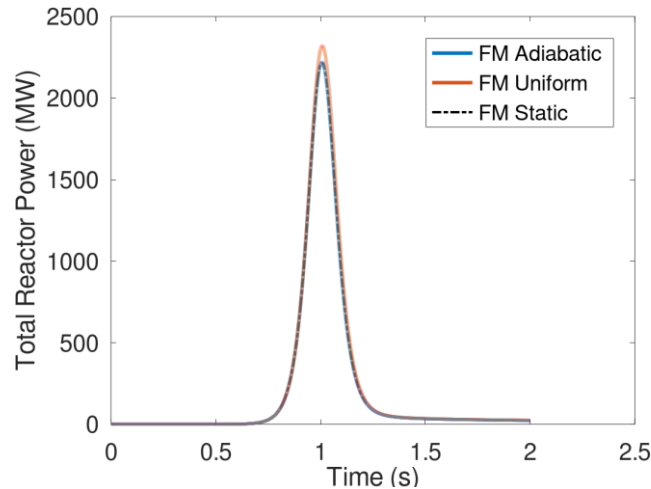
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Transients (Jake Eichenlaub)



Transient Fission Matrix in TREAT  
Reactor (Alvaro Pizarro-Vallejos)



# Transient fission matrix methods have been investigated for the TREAT reactor



# Thank you!

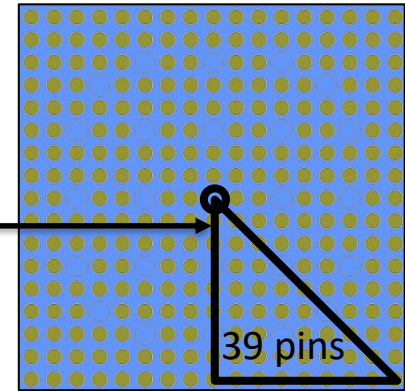
# Questions?



# Backup Slides

# The time of the RAPID database calculation is summarized

Calculation	Model Description		Number of Variations	Time (CPU-hr)	
	Geometry	Source		per Var.	Total
$\tilde{a}_{i,j}$ FM	1) Inf. assemblies	Single pin	7×39	0.45	121.9
Refl. correction	2) Standard core	Criticality	1	3.8	156.2
	3) Inf. core	Fixed from (2)	1	50.5	
	4) Inf. radial core	Fixed from (2)	1	50.7	
	5) Inf. axial core	Fixed from (2)	1	51.2	
Enrich. correction	6) Four-assembly	Uniform	2 (interp)	3.3	6.6
	7) Four-assembly	Uniform	3×3 (exact)	3.3	29.7
BA correction	8) Four-assembly	Uniform	4×16	3.3	52.8
Local correction	9) Four-assembly	Criticality	17	50	850
Total (exact $R$ )					337.5
Total (interp. $R$ )					314.4
Total (local. $R$ )					1128



# RAPID result is compared to the Serpent 2 reference calculation

		Uncertainty (Serpent) and relative error (RAPID)					Wall-clock
		2D pin-wise			3D (100 axial levels)		Time
Method	$k_{eff}$	$k_{eff}$	RMS	MAX	RMS	MAX	(20 cores)
Serpent	1.00402	0.5 pcm	0.18%	0.43%	1.86%	11.35%	80 hr
RAPID (old)	1.00245	-156 pcm	6.26%	22.27%	6.63%	57.53%	2.4 min
RAPID (interp.)	1.00428	26 pcm	0.75%	2.88%	2.30%	54.96%	2.4 min
RAPID (exact)	1.00427	25 pcm	0.54%	2.39%	2.26%	54.29%	2.4 min
RAPID (local)	1.00411	9 pcm	0.60%	2.11%	2.28%	54.51%	2.4 min

Note:  $1 \text{ pcm} = 10^{-5} k_{eff}$

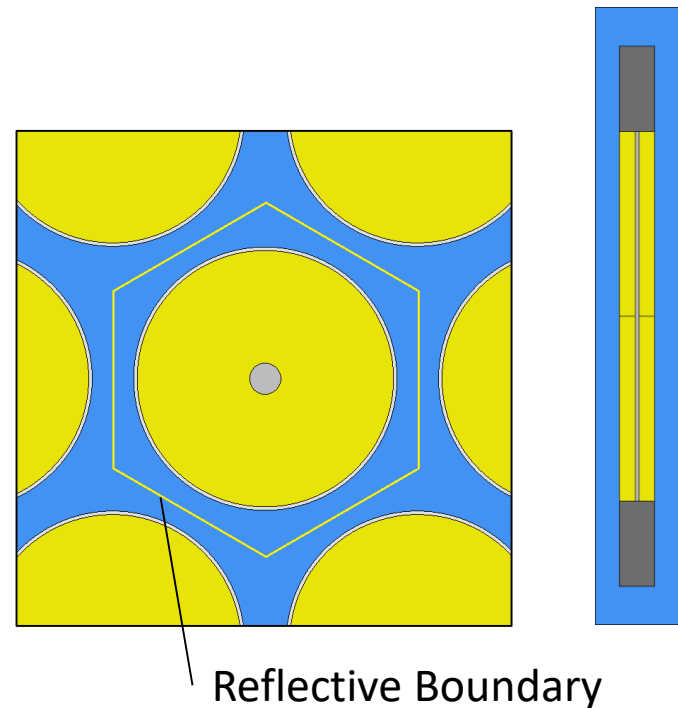
Pre-calculation time:

- Exact : 16.8 hours
- Interpolated: 15.7 hours
- Localized: 56.4 hours

# Test Problems: Problem Geometry

15

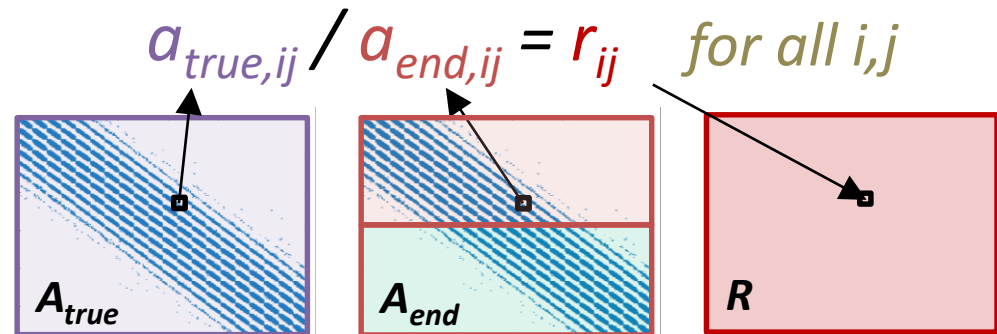
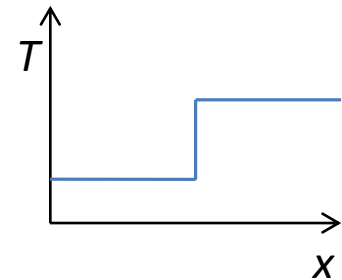
- Used Serpent 2.1.30 for Monte Carlo
- Single fuel pin
  - Graphite reflector at ends
  - Fresh 8.5% U-ZrH<sub>1.6</sub> fuel
- Boundary conditions:
  - Reflective in X-Y
  - Vacuum in Z
- Vary axial fuel temperature
- Tally fission source in 128 axial bins



# Method: Ratio-Correction in Present Work

8

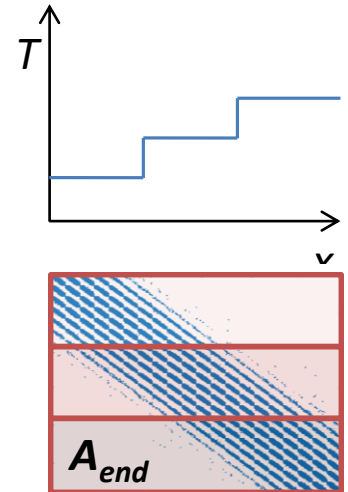
- Pre-calculation:
  - Simulate one additional, non-uniform case
  - Obtain true fission matrix  $\mathbf{A}_{true}$  from (Monte Carlo) criticality calculation
  - Construct  $\mathbf{A}_{end}$
  - Calculate matrix  $\mathbf{R}$



# Method: Ratio-Correction in Present Work

9

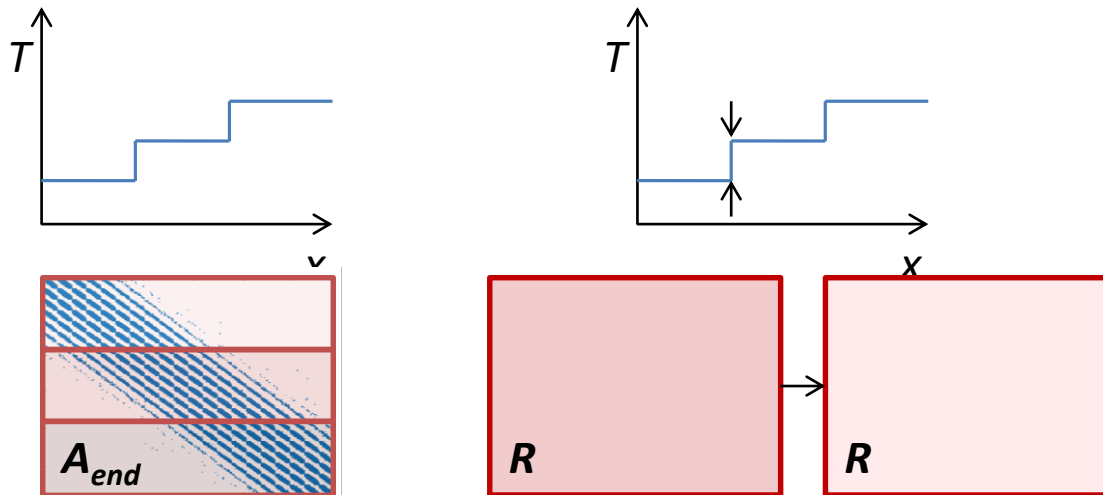
- Apply to arbitrary temperature distribution:
  - Superimpose correction matrix  $\mathbf{R}$  for each temperature change in arbitrary profile
- Application assumes:
  - Geometric similarity ( $\mathbf{R}$  independent of position)
  - $\mathbf{R}$  scales linearly with temperature



# Method: Ratio-Correction in Present Work

10

- 1. Scale  $R$  by temperature



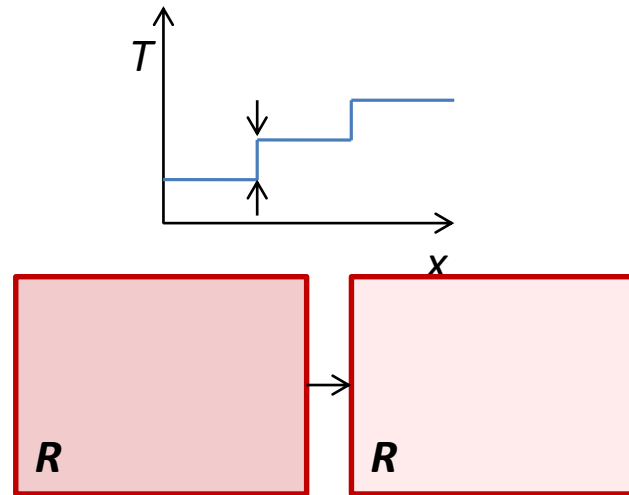
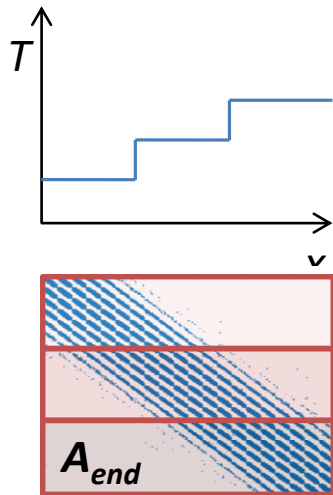
Scale  $R$  for temperature



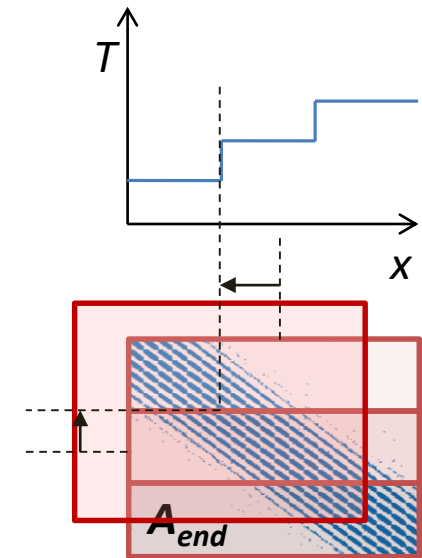
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11

- 2. Translate  $R$  and multiply



Scale  $R$  for temperature

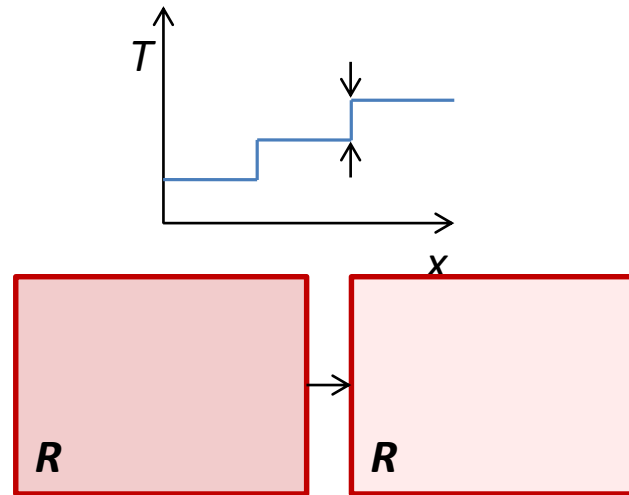
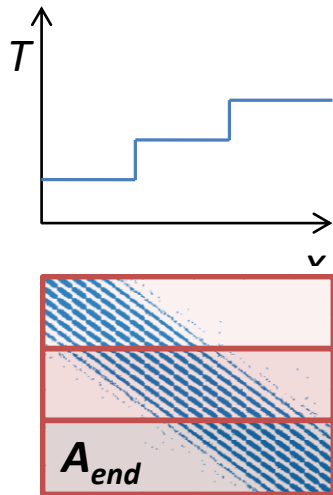


Translate  $R$  and multiply

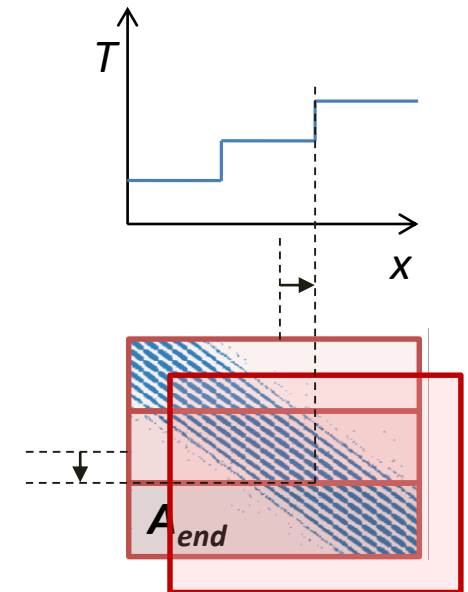
# Method: Ratio-Correction in Present Work

12

- 3. Repeat for all temperature changes



Scale  $R$  for temperature

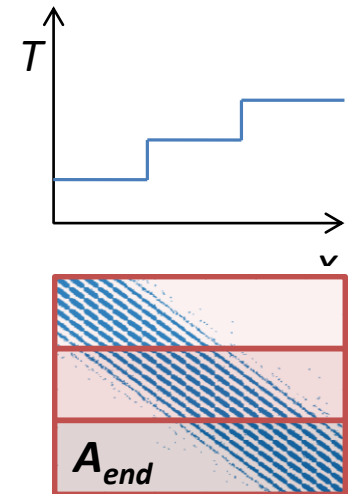


Translate  $R$  and multiply

# Method: Ratio-Correction in Present Work

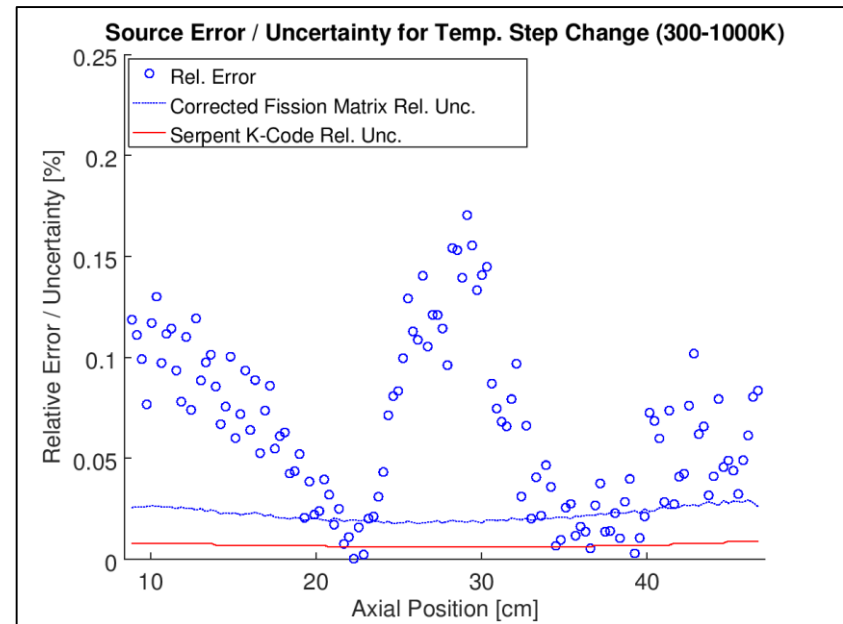
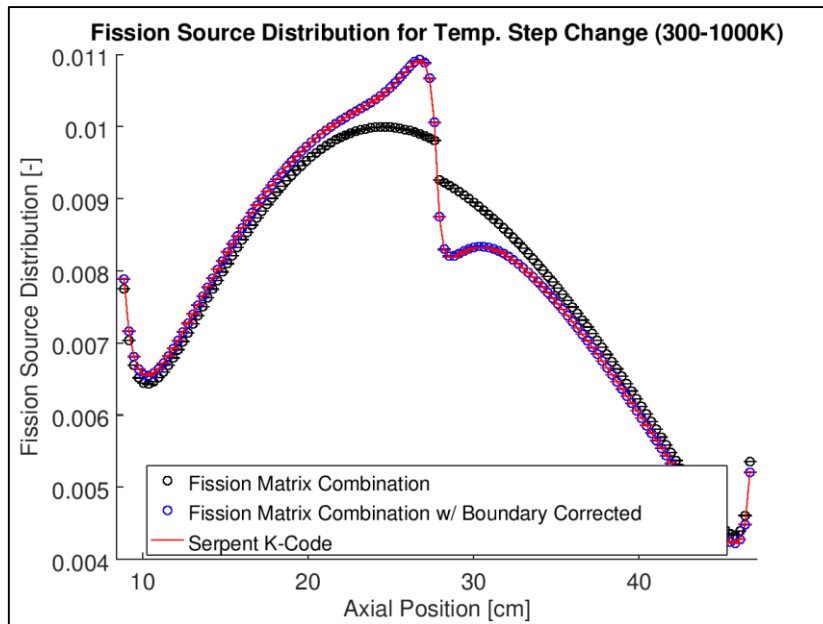
13

- Apply to arbitrary temperature distribution:
  - Superimpose correction matrix  $R$  for each temperature change in arbitrary profile
- Application assumes:
  - Geometric similarity ( $R$  independent of position)
  - $R$  scales linearly with temperature



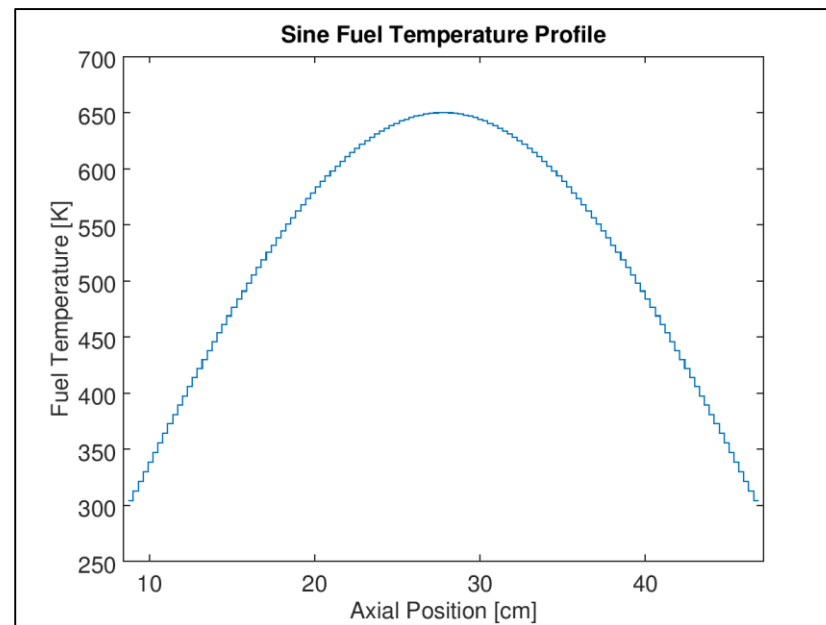
# Results: Step Temperature Profile

29



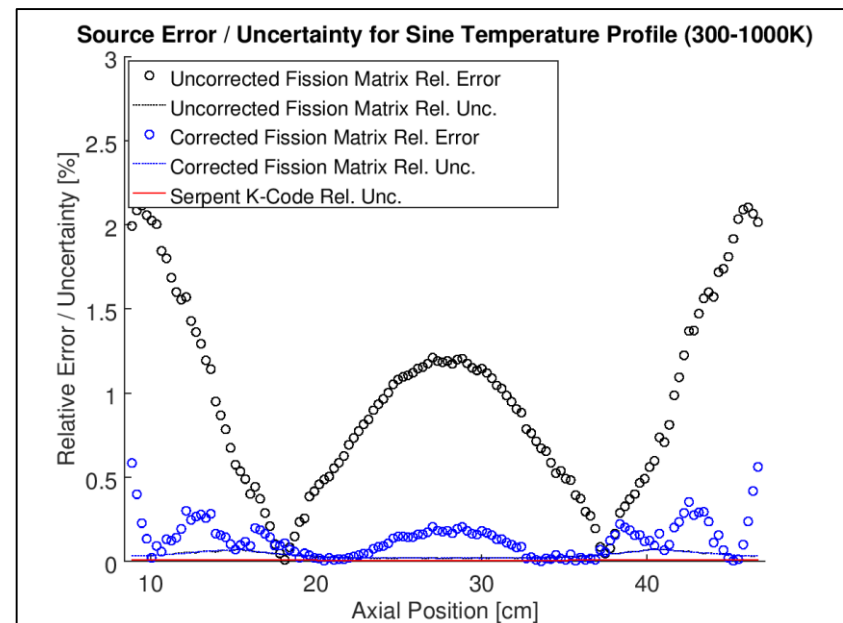
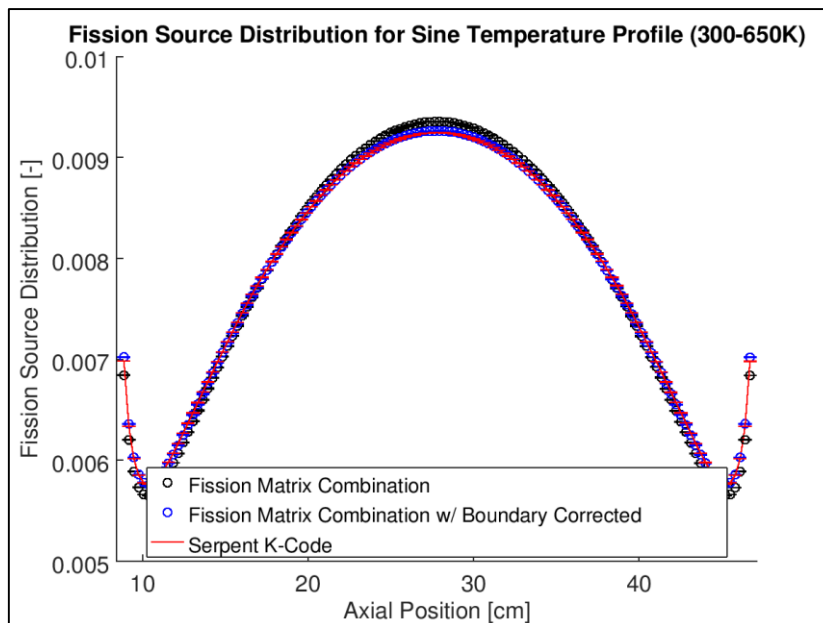
# Results: Sine Temperature Profile

31



# Results: Sine Temperature Profile

32



# Results: Sine Temperature Profile

33

- End method produces reasonable estimate of  $k_{eff}$
- Correction reduces source RMS error by factor of 5
- Calculation takes ~4 seconds on personal laptop

	$K_{eff}$ Relative Error [pcm]	Source RMS Rel. Error [%]	Source Max. Rel. Error [%]
End Method	-41	1.1	-2.11
Corrected End Method	-39.6	0.17	0.58

# Results: Sine Temperature Profile

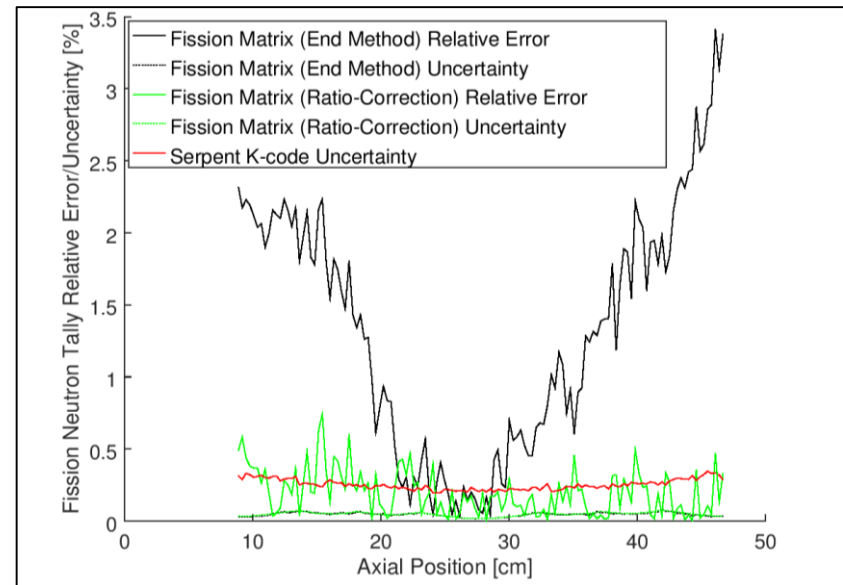
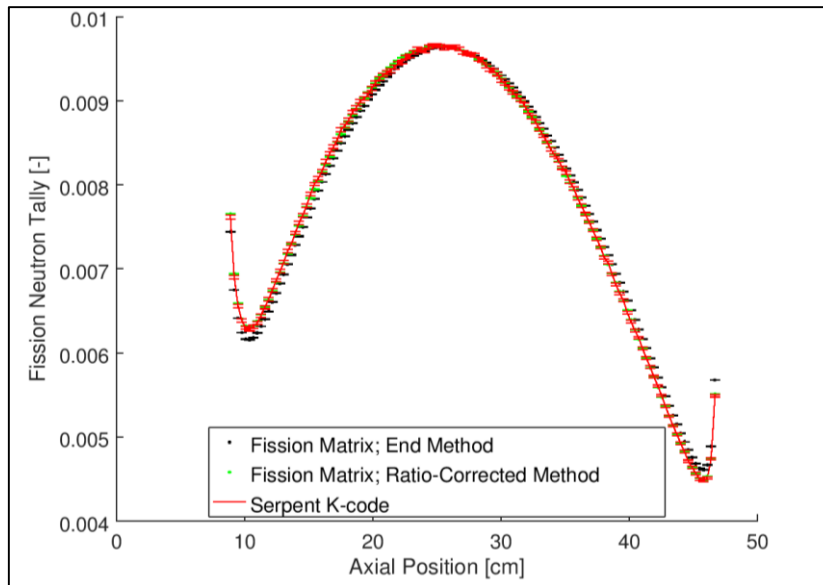
34

Temp. Range [K]	$k_{eff}$ RE [pcm]	Src. RMS RE [%]	Src. Max. RE [%]
<b>End Method</b>			
300-1000	-6.4	2.41	-4.9
300-650	-41	1.10	-2.11
300-475	-53.1	0.47	-0.9
650-1000	20.5	1.21	-2.82
<b>Corrected End Method</b>			
300-1000	7.4	0.40	-1.01
300-650	-39.6	0.17	0.58
300-475	-53.8	0.14	0.47
650-1000	23.2	0.17	-0.47



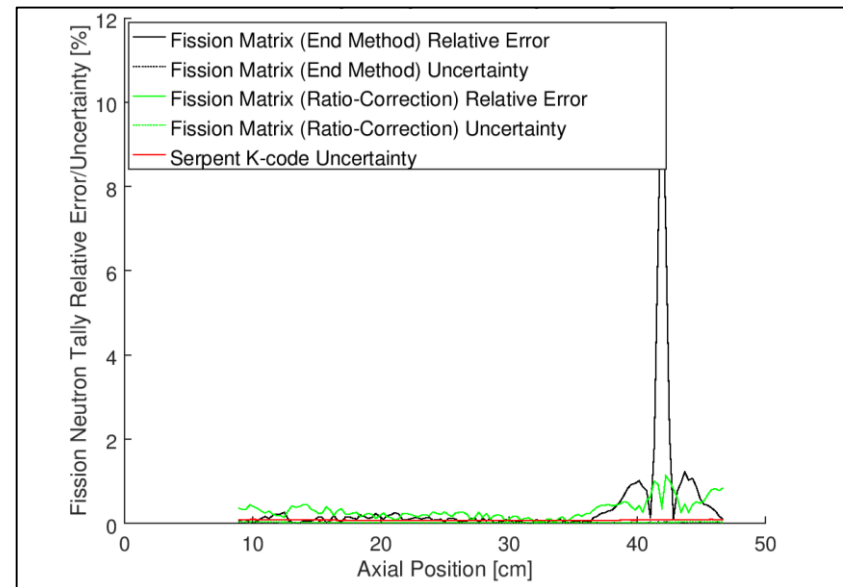
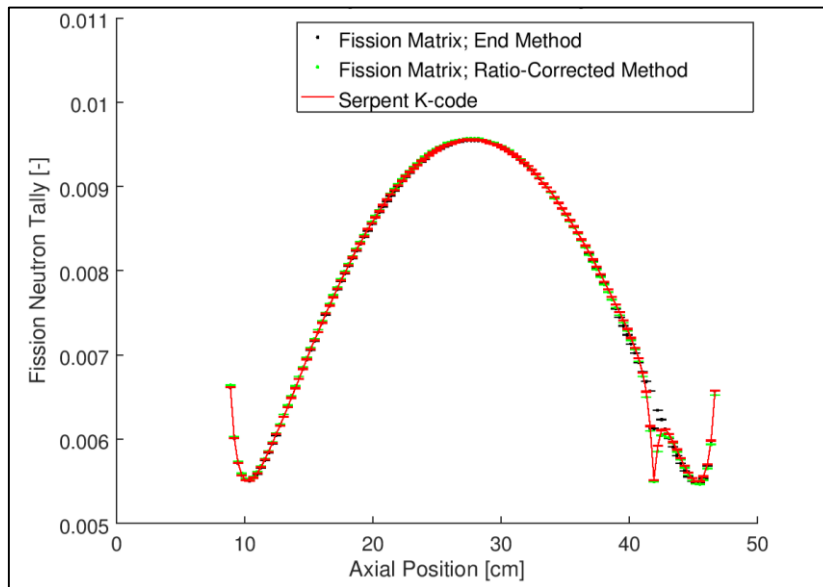
# Linear Temperature Profile

Linearly increasing 300K to 1000K from bottom to top



# “Dirac” Temperature Profile

Uniform 300K with one “cell” elevated to 1000K



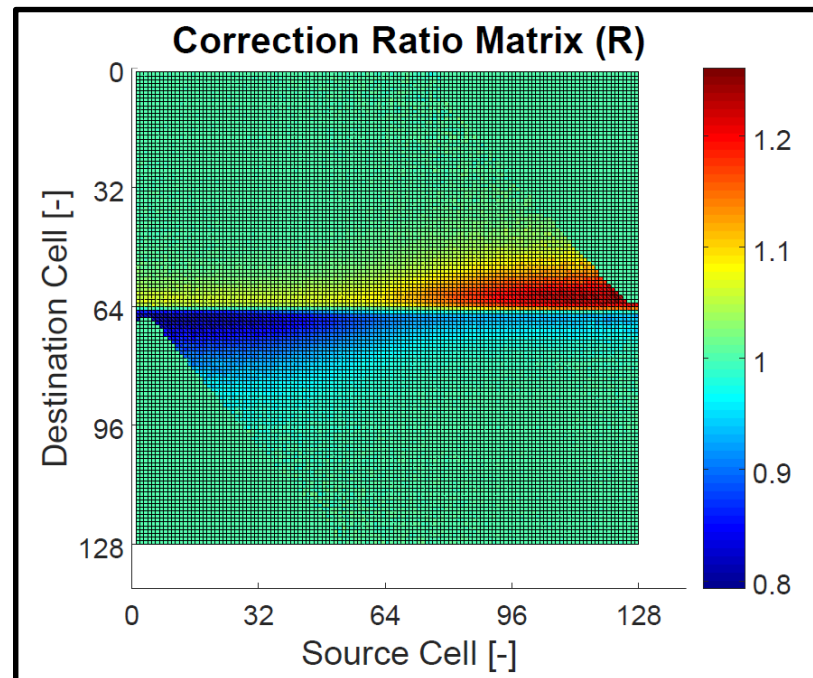
# Additional Temperature Profiles

Profile - Method [-]	$k_{eff}$ RE [pcm]	Src. RMS RE [%]	Src. Max. RE [%]
Linear - End	-88.9	1.58	3.41
Linear - Corrected	-68.4	0.25	-0.74
Dirac - End	8.4	1.38	11.09
Dirac - Corrected	-26.4	0.37	-1.13

# Results: Step Temperature Profile

24

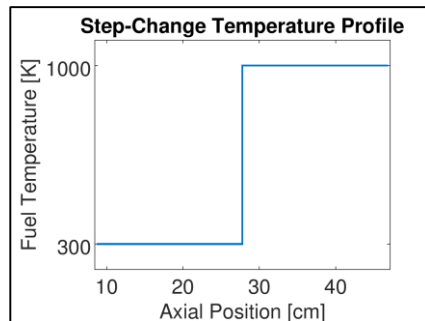
- Calculate ***R***:



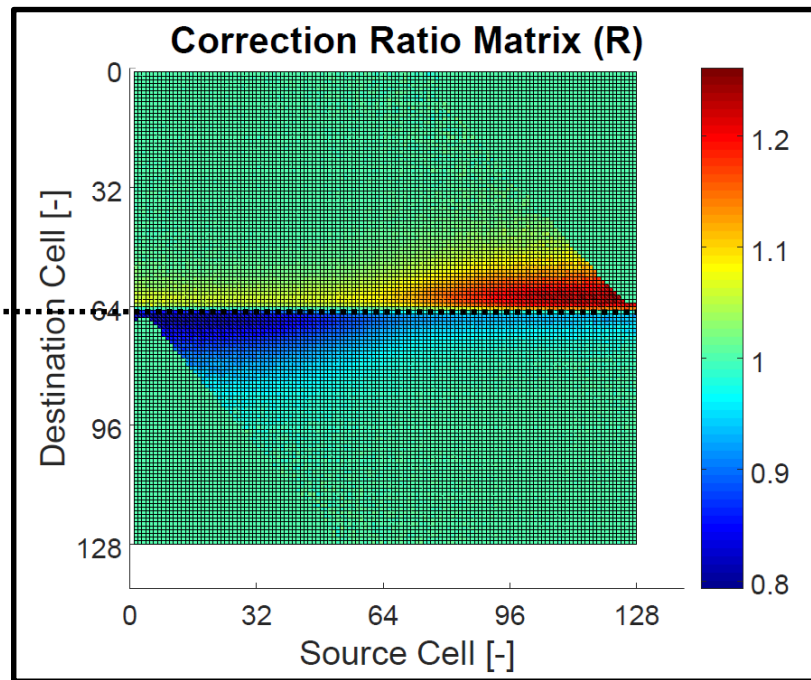
# Results: Step Temperature Profile

25

- Calculate ***R***:



Temperature  
step change



Most entries near  
 $\sim 1$

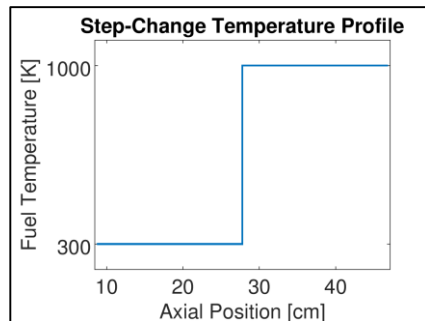
Entries with  
uncertainty >1% set  
to 1

Contribution from  
high-temperature  
cells increases and  
vice versa

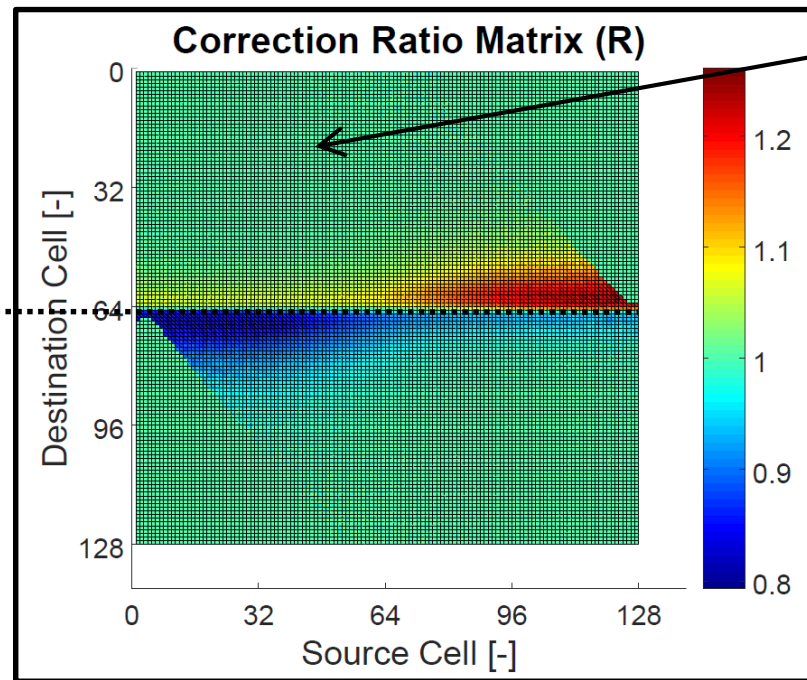
# Results: Step Temperature Profile

26

- Calculate ***R***:



Temperature  
step change



Most entries near  
~1

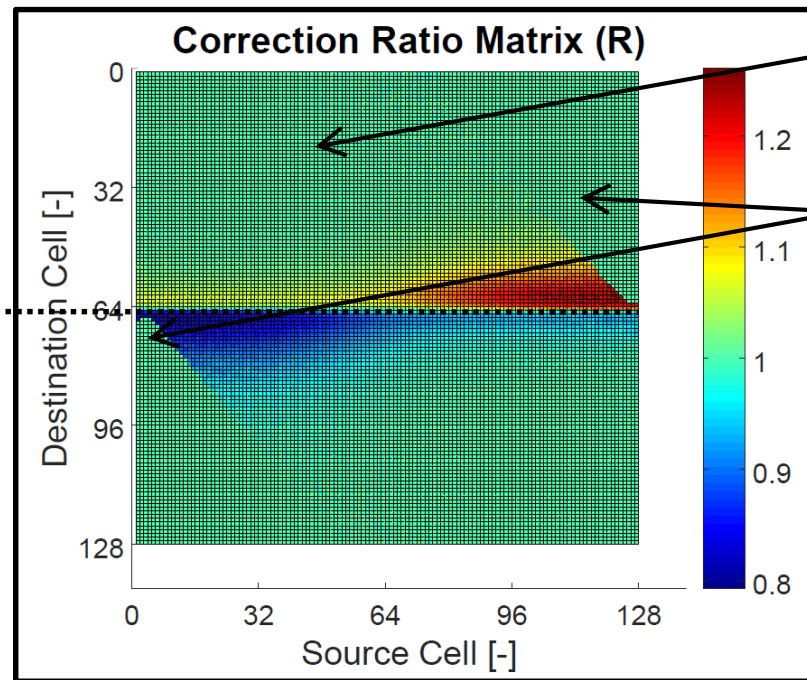
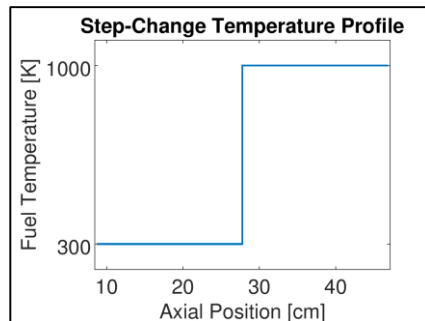
Entries with  
uncertainty >1% set  
to 1

Contribution from  
high-temperature  
cells increases and  
vice versa

# Results: Step Temperature Profile

27

- Calculate ***R***:



Most entries near ~1

Entries with uncertainty >1% set to 1

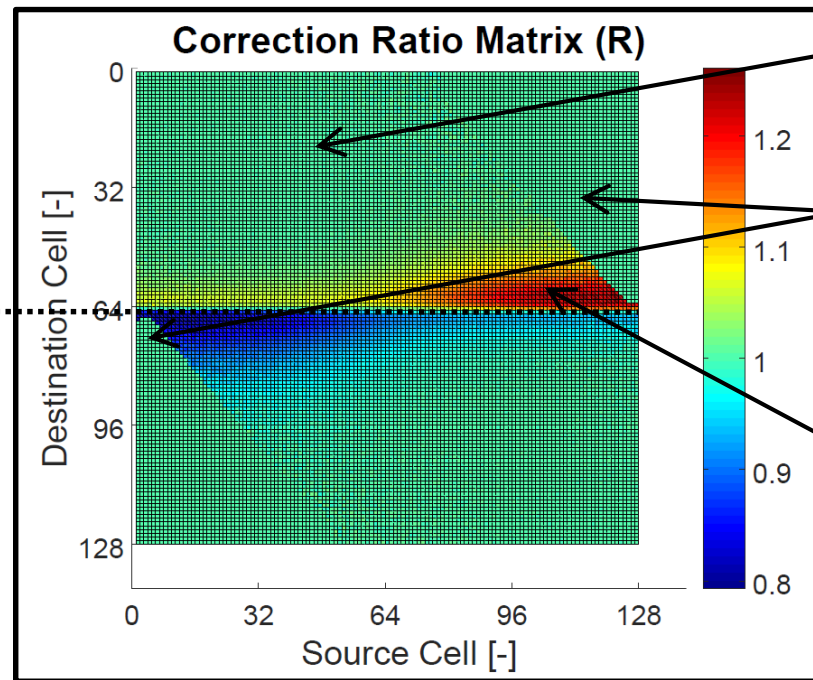
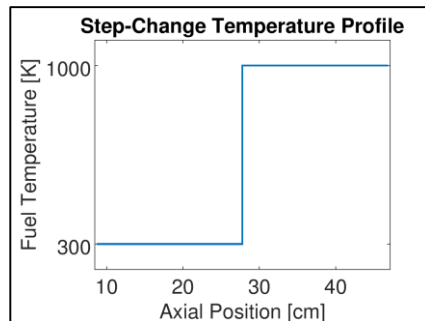
Contribution from high-temperature cells increases and vice versa



# Results: Step Temperature Profile

28

- Calculate ***R***:



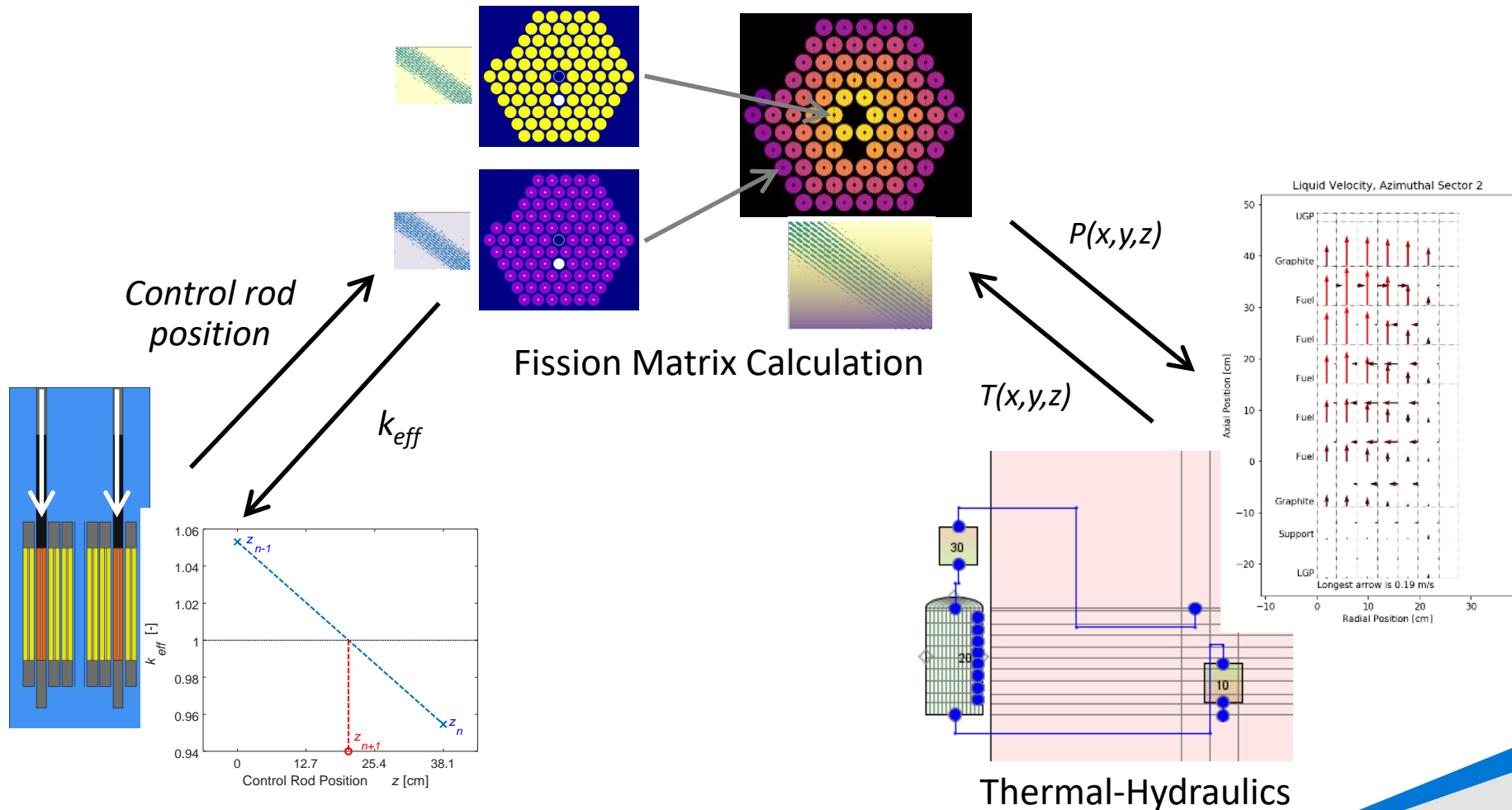
Most entries near ~1

Entries with uncertainty >1% set to 1

Contribution from high-temperature cells increases and vice versa



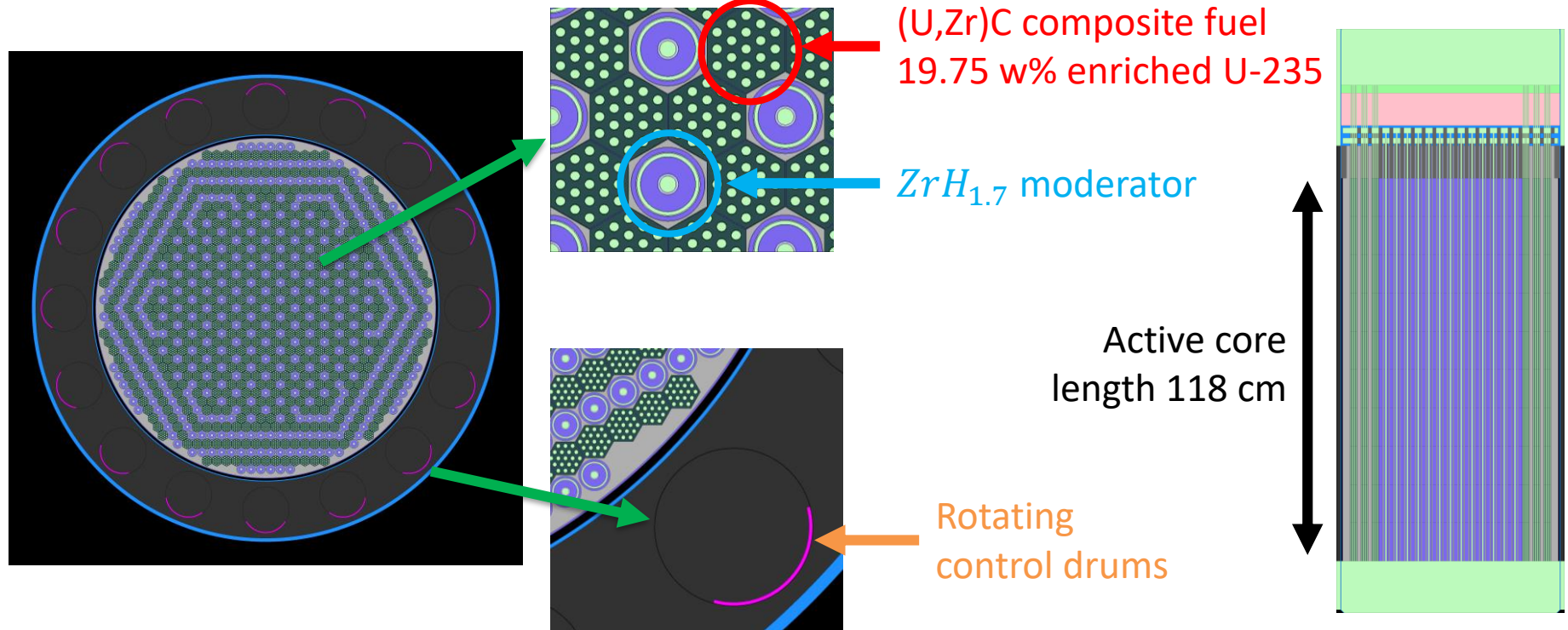
# Steady-state temperatures / control rod positions were found by iterating with TRACE



# After pre-calculation, calculations are completed in seconds/minutes

Fission Matrix Database Calculation (one calculation)	80 CPU-hrs
Fission Matrix Database Calculation (total)	2400 CPU-hrs
Fission Matrix Assembly and Solution (per fission matrix)	0.4 seconds
Uncertainty Calculation (per fission matrix)	6.2 seconds
Error Calculation / Printing / Plotting (per fission matrix)	5.4 seconds
Reading FM database and fitting polynomials (per batch)	41 seconds

# The same method can be applied to Nuclear Thermal Propulsion type reactors

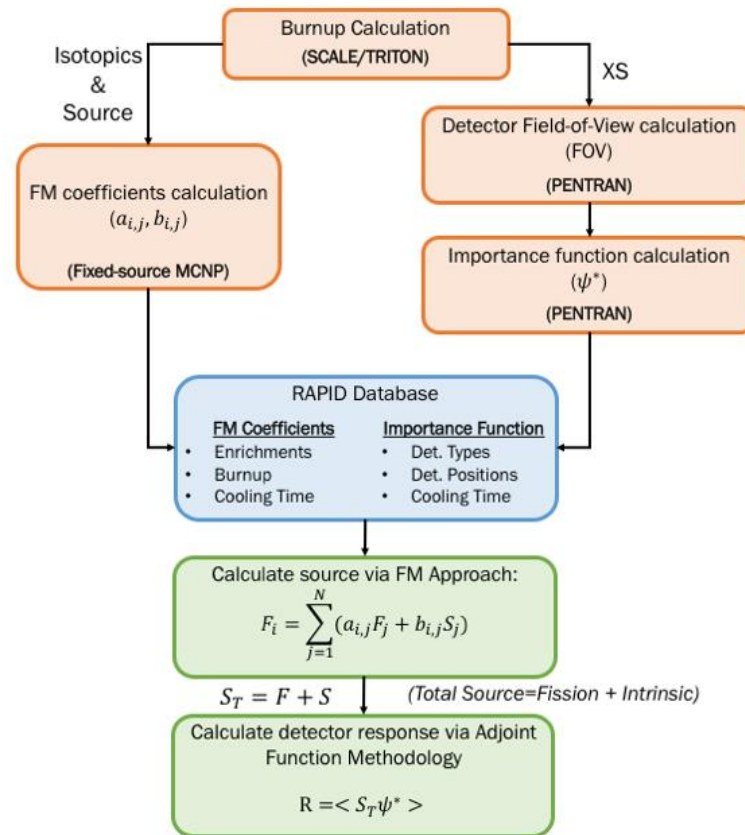
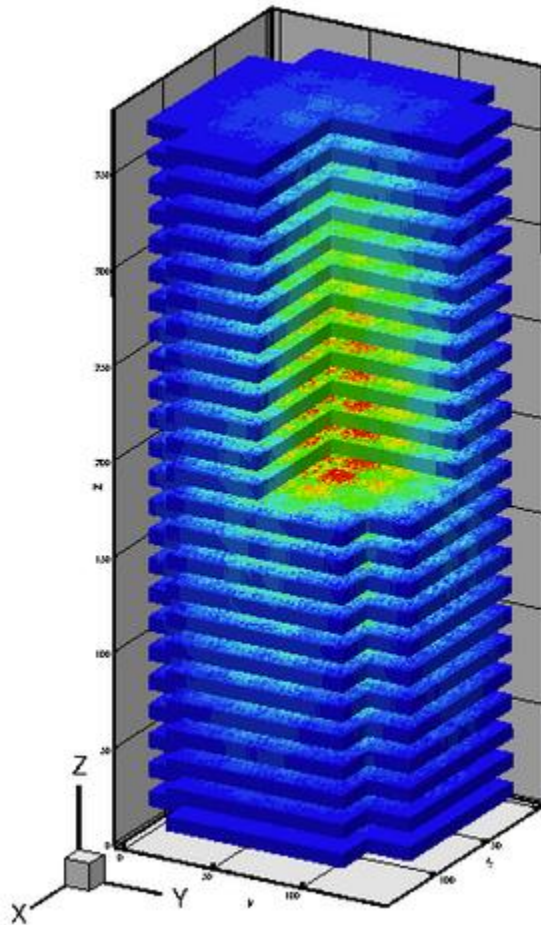


Both are small cores, hexagonal loading patterns  
Control geometry is different (drums vs. fuel-followed rods)  
Fuel/Moderator Heterogeneity and Feedback Mechanisms are different

# Presentation Outline

- RAPID Material Correction Ratios (Donghao He)
- Temperature/Control Methods for TRIGA and NTP (Adam Rau)
- Xenon Transients in Penn State TRIGA Reactor (Jake Eichenlaub)
- Transient Fission Matrix Methods in TREAT Reactor (Alvaro Pizarro-Vallejos)

# RAPID uses fission matrix methods for fast calculations



# Fission Matrix Method

2. Fit polynomial coefficients to fission matrix database

$$a_{ij}^{(n_T n_\theta)} = \sum_{k=0}^K \sum_{l=0}^L c_{ijkl} (T^{(n_T)})^k (\cos(\theta^{(n_\theta)}))^l$$

$$A_{ij} = C_{ij} V$$

$$C_{ij} = A_{ij} V^T (V V^T)^{-1}$$

# Control Drum Angle: Summary

Control Drum Angle [deg]	$k_{\text{eff}}$ Error [pcm]	Norm. RMS Fiss. Src. Error [%]	Norm. Max. Fiss. Src. Error [%]
0	5.2	0.67	-2.29
8	9.7	0.60	-2.54
25	3.8	0.54	-2.17
38	-1.3	0.57	-1.87
53	0.1	0.58	2.01
68	17.3	0.68	-2.67
83	3.4	0.59	-2.22
98	12.4	0.63	-2.88
113	-7.0	0.65	-2.27
123	-22.0	0.71	-2.38
126	7.4	0.57	-2.35
128	3.1	0.65	-2.58
129	-6.8	0.55	-2.51
132	-5.8	0.65	-2.44
143	-10.6	0.53	-2.11
153	-11.8	0.68	-2.80
156	-7.4	0.66	-2.39
158	-19.5	0.53	-2.88
159	-14.6	0.62	-2.03
162	3.4	0.56	-2.03
173	-8.6	0.63	-2.20
180	-13.9	0.69	-2.59

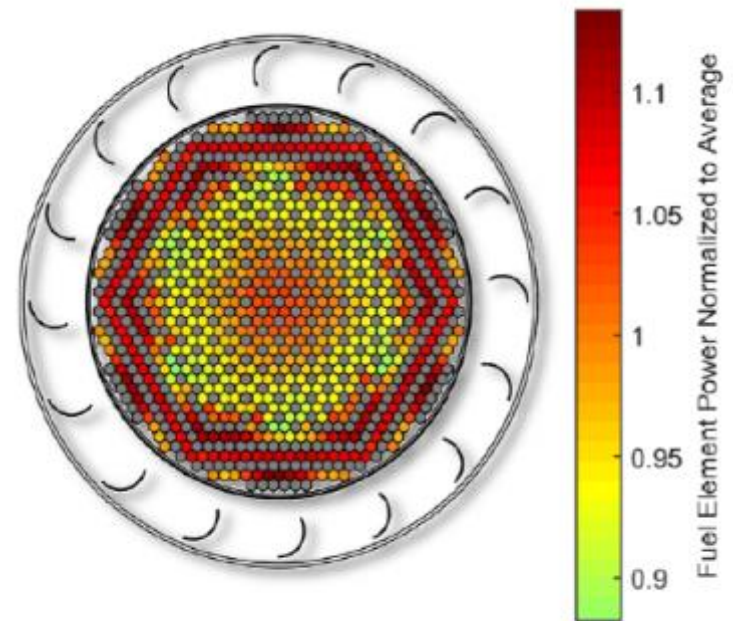
# Case: Temperature Profile

- Inlet:
  - Uniform 300K
- Outlet:
  - Proportional to lateral peaking factor at 300K
  - Max. temp. controlled as parameter
- Linearly increasing along axis



# Case: SULEU Core

- Superb Use of Low-Enriched Uranium (SULEU) Core (Venneri et al., 2016)
- Based on heritage NERVA design
  - Demonstrate feasibility of LEU
  - Retains graphite-composite fuel elements and other peripherals
- Meets NASA DRA 5.0 requirements
  - Mass: 2,498 kg
  - Thrust: 35,000 lb<sub>f</sub>
  - $T_{max}$ : 2,850 K
  - Specific Impulse: 898 s



*Venneri et al., 2016*

# Cases: Test Cases

- Fuel Temperature Cases:
  - Max. Fuel Temperature set to 400, 800, 1200, 1600, 2000 K
  - Control drums fully withdrawn
- Control Drum Cases:
  - Control drums set to 0, 8, 25, 38, 53, 68, 83, 98, 113, 123, 126, 128, 129, 132, 143, 153, 156, 158, 159, 162, 173, 180 degrees
  - Max. Fuel Temperature fixed at 2000 K

# Cases: Test Cases

- Varied max. temperature and control drum angle separately (no case has uniform temperature)

**Varied control drum angle**



**Varied temperature**



Max. Fuel Temp. [K]	Control Drum Angle [deg.]	Max. Fuel Temp. [K]	Control Drum Angle [deg.]
2000	0	2000	128
2000	8	2000	129
2000	25	2000	132
2000	38	2000	143
2000	53	2000	153
2000	68	2000	156
2000	83	2000	158
2000	98	2000	159
2000	113	2000	162
2000	123	2000	173
2000	126	2000	180

Max. Fuel Temp. [K]	Control Drum Angle [deg.]
400	0
800	0
1200	0
1600	0
2000	0

# Previous Work: Steady-State

- Rau and Walters 2018, Topham et al. 2019
  - Coupled TRACE/Fission Matrix simulation of TRIGA reactor
  - Fission matrix method used to investigate sensitivities in TRACE and validate on initial startup data

# Features of Fission Matrices

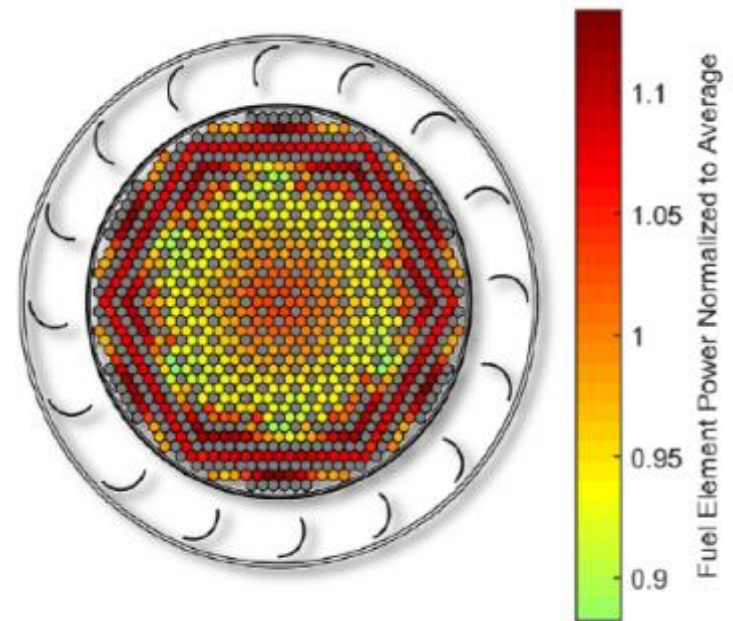
$$k_{eff} \vec{s} = \mathbf{A} \vec{s}$$

$\vec{s}$  is eigenvector of  $\mathbf{A}$

$k_{eff}$  is eigenvalue of  $\mathbf{A}$

# Case: SULEU Core (cont.)

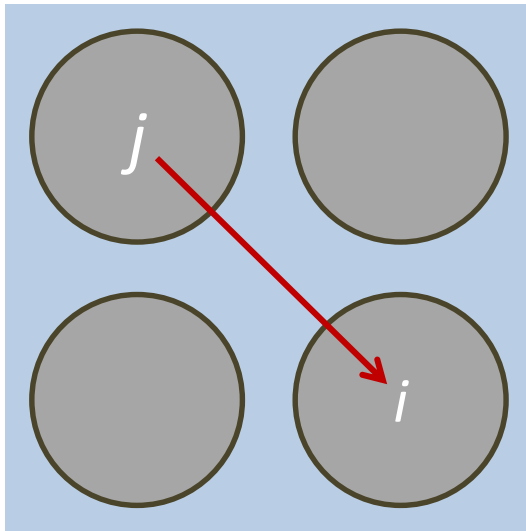
- (U,Zr)C composite fuel
- $\text{ZrH}_{1.7}$  moderator
- 66 cm core diameter (~26 in.)
- 16 rotating control drums on core periphery



*Venneri et al., 2016*

# Features of Fission Matrices

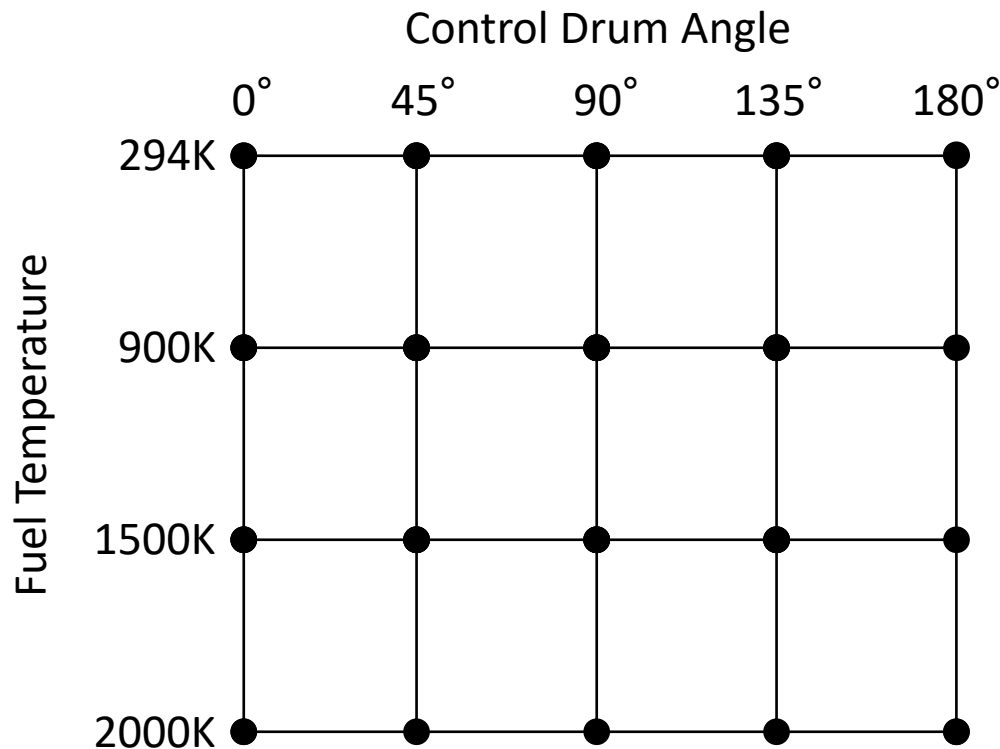
$$a_{ij} = f(\vec{r}_j \rightarrow \vec{r}_i)$$



Elements of  $\mathbf{A}$  ( $a_{ij}$ ):  
expected number of  
neutrons induced in  
cell  $i$  from a neutron  
born in cell  $j$

# Methodology: Preprocessing

1. Serpent used to pre-calculate fission matrices at uniform fuel temperature and control drum angle



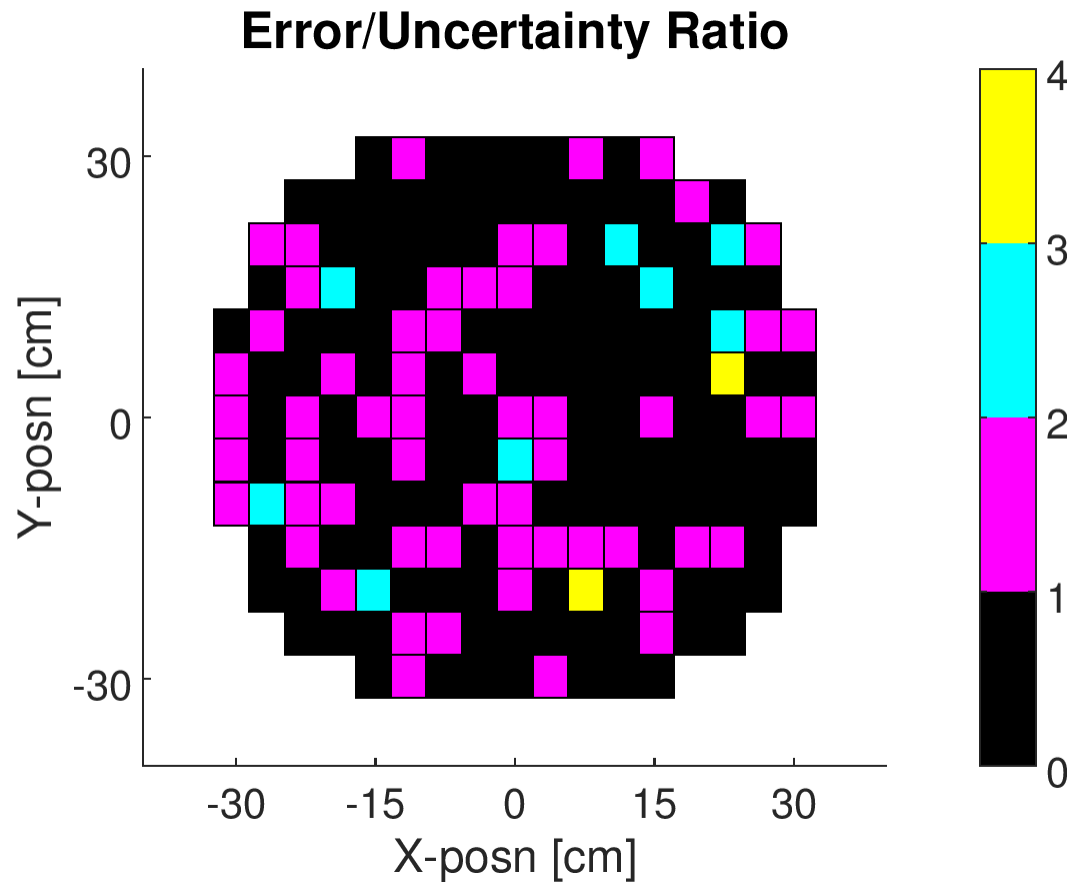
4 fuel temperatures  
× 5 control drum angles  

---

20 database calculations



# Fuel Temperature: Error/Uncertainty



# Normalized Error

$$\hat{e}_i = \frac{f_{i,FM} - f_{i,serp}}{\left(\frac{1}{n}\right) \sum_i^n f_{i,serp}}$$

# Margins



