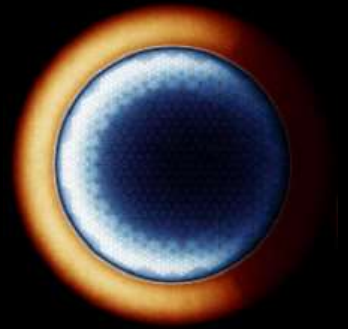

FULL CORE POWER AND ISOTOPIC OSCILLATIONS WITH VARIOUS DEPLETION SCHEMES

ANDREW JOHNSON

COMPUTATIONAL REACTOR ENGINEERING

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OUTLINE

- Governing depletion equations
- Summary of depletion coupling schemes
- Full core analysis
- Single assembly convergence study
- Conclusions
- Discussion

GOVERNING EQUATIONS

- Bateman equations (Bell and Glasstone 1970)

$$\frac{dN(r,t)}{dt} = \underline{\underline{M}}\big(R(r,t)\big)N(r,t)$$

- $R(r,t)$ contains the reaction rates
- $\underline{\underline{M}}$ is a matrix describing transmutation of isotopes
- Can solve for $N(r,t)$ by taking exponent of matrix

- Transmuation trajectory analysis (TTA) (Cetnar 2006)
- Chebyshev rational approximation method (CRAM) (Pusa and Leppänen 2010; Lago and Rahnema 2017; Pusa 2011)
- Reaction rates and nuclide distributions are strongly coupled
- Temperature dependence through Doppler broadening

SOLUTION METHODS - OVERVIEW

Predictor - CE

1. Beginning of step (BOS) flux depletes to obtain N_{i+1}

Predictor-corrector – CE/LI

1. BOS flux depletes to obtain N_{i+1}^{P}
2. Predictor end-of-step (EOS) number density is used to obtain a corrector BOS flux ϕ_i^C
3. Average of fluxes is used to deplete across interval and obtain EOS isotopics

$$\overline{\phi_i} = \frac{1}{2}\left(\phi_i^P + \phi_i^C\right)$$

$$\overline{\phi_i} \rightarrow N_{i+1}$$

SOLUTION METHODS - OVERVIEW

Higher order methods utilize previous EOS flux (Isotalo and Aarnio 2011a)

- Extrapolate using ϕ_{i-1} and ϕ_i to obtain flux throughout depletion interval ϕ_i^P
- Deplete with ϕ_i^P and obtain N_{i+1}^P
- Set the next BOS number density using interpolation

$$N_{i+1} = w_{i-1}N_{i-1} + w_i N_i + w_{i+1}N_{i+1}^P$$

- Linear extrapolation/Quadratic interpolation – LE/QI

Substep methodology (Isotalo and Aarnio 2011b)

- Assume that reaction rates are piecewise constant
- Many depletion sub steps inside depletion interval
- No additional transport solutions

SOLUTION METHODS - OVERVIEW

Stochastic Implicit Euler (SIE) (Dufek, Kotlyar, and Shwageraus 2013)

- Multiple transport solutions per depletion interval
  - Total histories equally distributed
- Iteration-wise EOS flux obtained using iteration EOS number density

$$N_i^{(n)} \rightarrow \phi_i^{(n)}$$

- Updated end of step flux is average of iteration-wise fluxes

$$\overline{\phi_i^{(n)}} = \frac{1}{n}\sum_{j=1}^{n} \phi_i^{(j)}$$

- Averaged flux used to deplete and obtain isotopics for next iteration

$$\overline{\phi_i^{(n)}} \rightarrow N_i^{(n+1)}$$

- After N iterations

$$\phi_{i+1}^{(0)} = \overline{\phi_i^{(N)}}$$

SOLUTION METHODS - SUMMARY

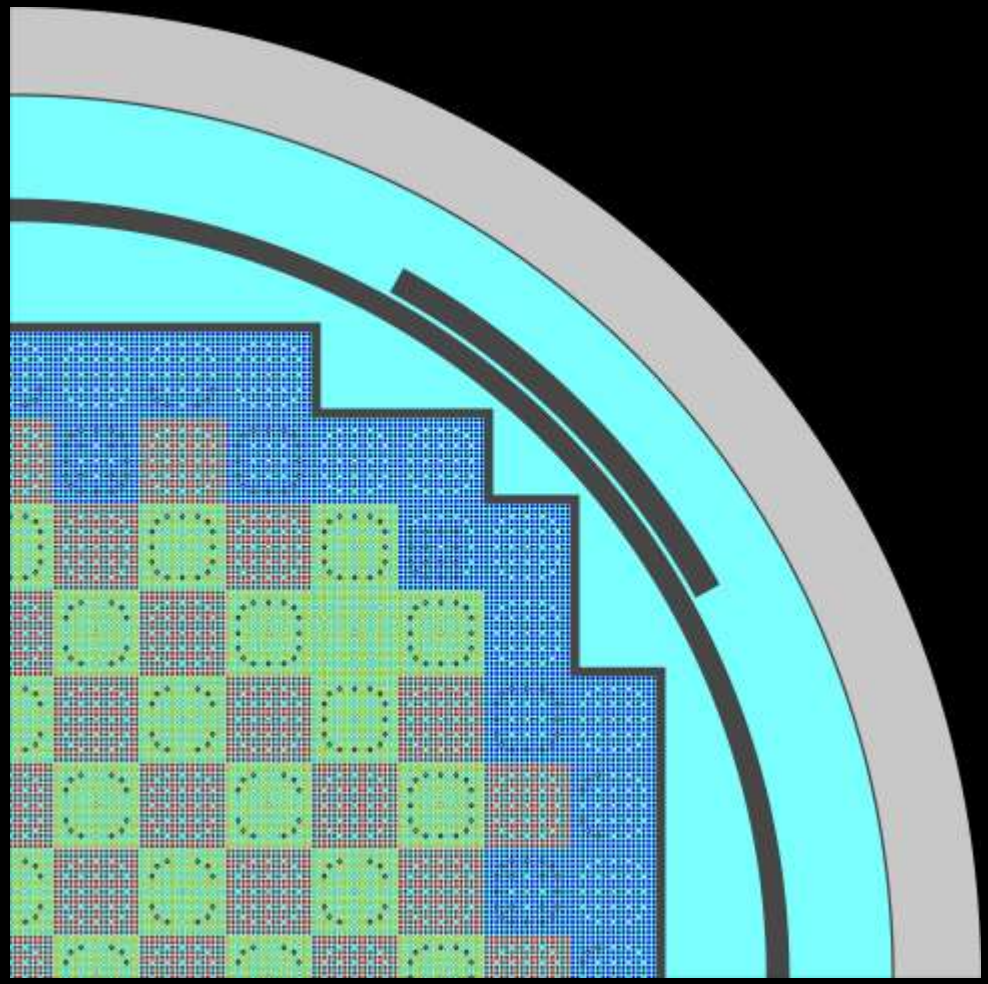
Explicit Euler variations

- Use flux ϕ_i to obtain the number density N_{i+1} separated by some Δt
- Assume that all other parameters are constant
  - Reaction rates
  - Reactor state
- Increasing Δt means fewer transport calculations
  - Accurate in the limit of small Δt

- Isotopic oscillations have been observed (Dufek et al. 2013; Kotlyar and Shwageraus 2013)
  - Multi-region problems

FULL CORE ANALYSIS

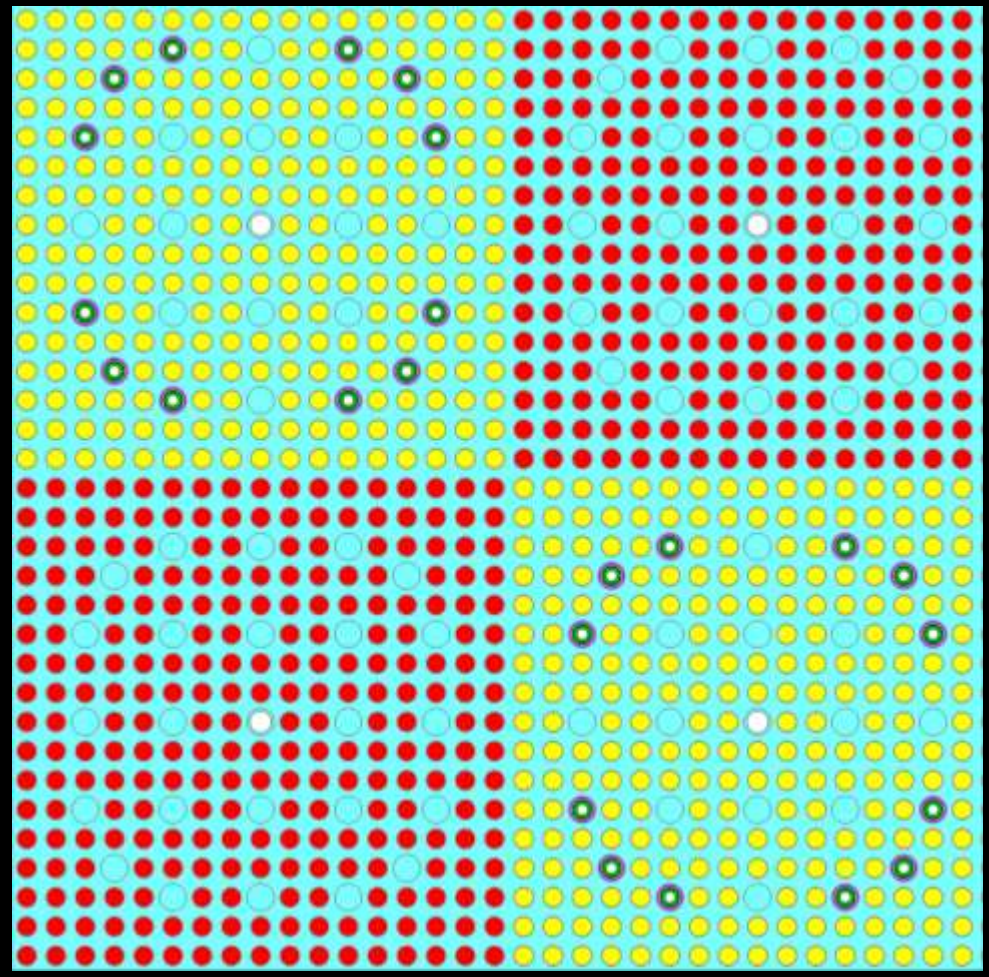
- BEAVRS reactor (Horelik et al. 2013)
- 150000 particles/cycle
- 100/500 inactive/active cycles
- ENDF/B-VII.0



Quarter of the BEAVRS v2 SERPENT Model

FULL CORE - MODELING

1. 1/8th core explicitly modeled
2. 12 axial layers of unique materials
3. Unique burnable materials per assembly
4. No spacer grids nor soluble boron



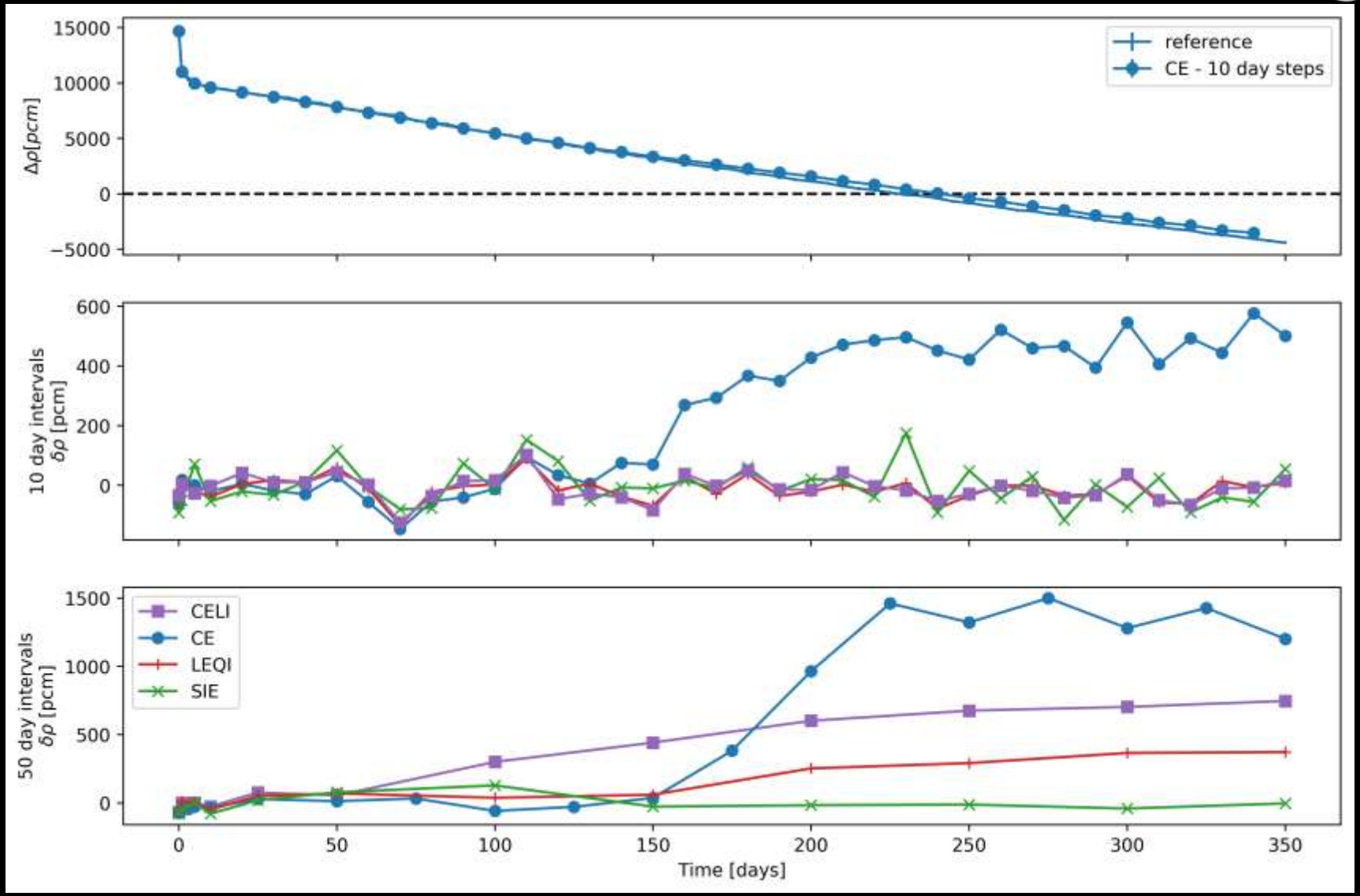
3.1 wt% (red) and 2.4 wt% (yellow) enriched assemblies

FULL CORE - DEPLETION

- 350 day depletion schedule
  - 1, 4, and 5 day initial depletion intervals
- Three separate depletion schemes
  - 5 day time steps
  - 10 day time steps
  - 50 day time steps
- Reference: SIE with 5 day time steps

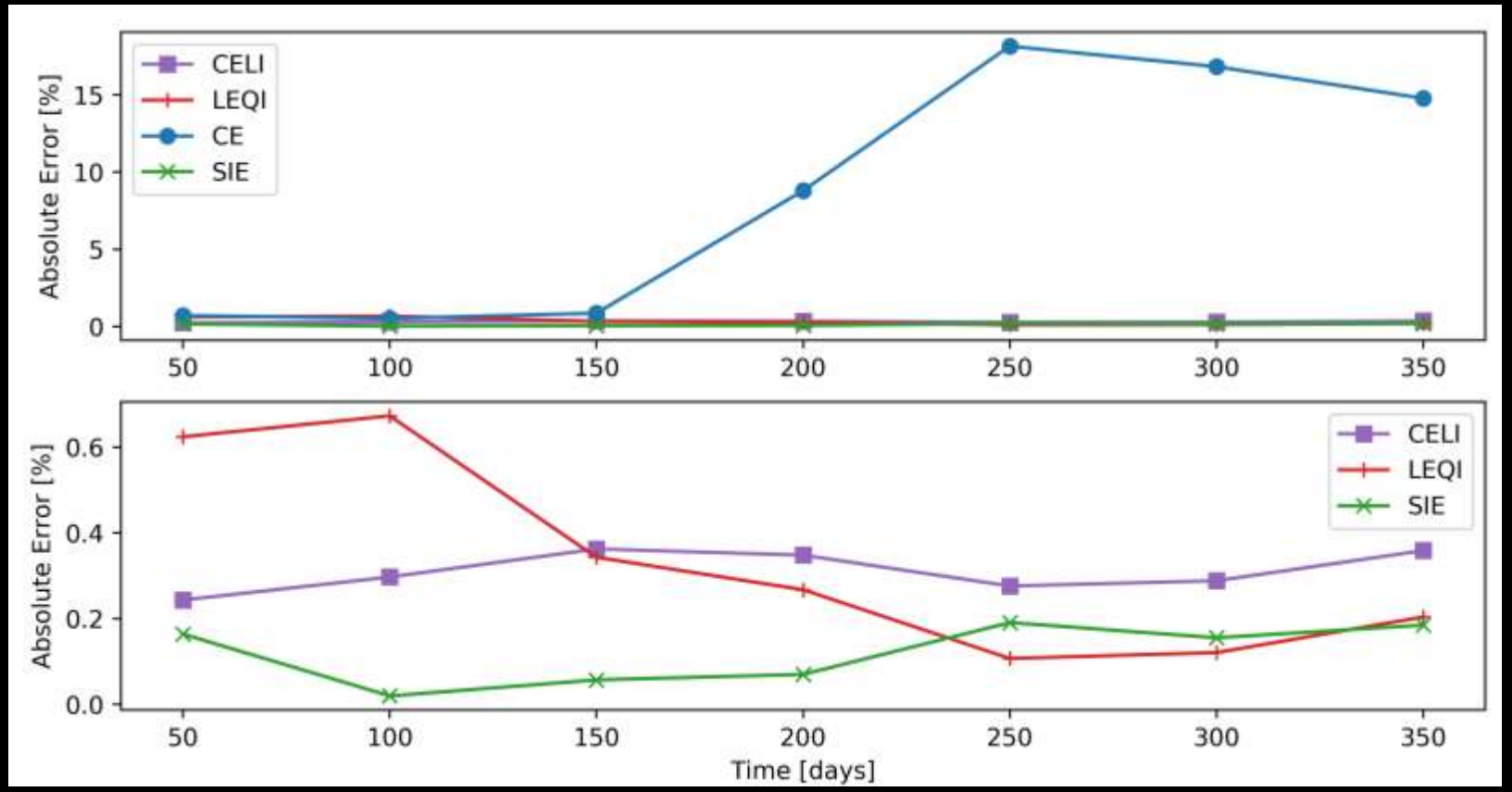
- Tested nearly all schemes offered by SERPENT (Leppänen et al. 2015)
  - CE, CE/LI, LE/QI, SIE
- SIE utilized 10 iteration per interval
  - $1/10^{th}$ as many particles per cycle
- CE used twice as many depletion events
  - Equal number of transport calculations

FULL CORE – KEFF



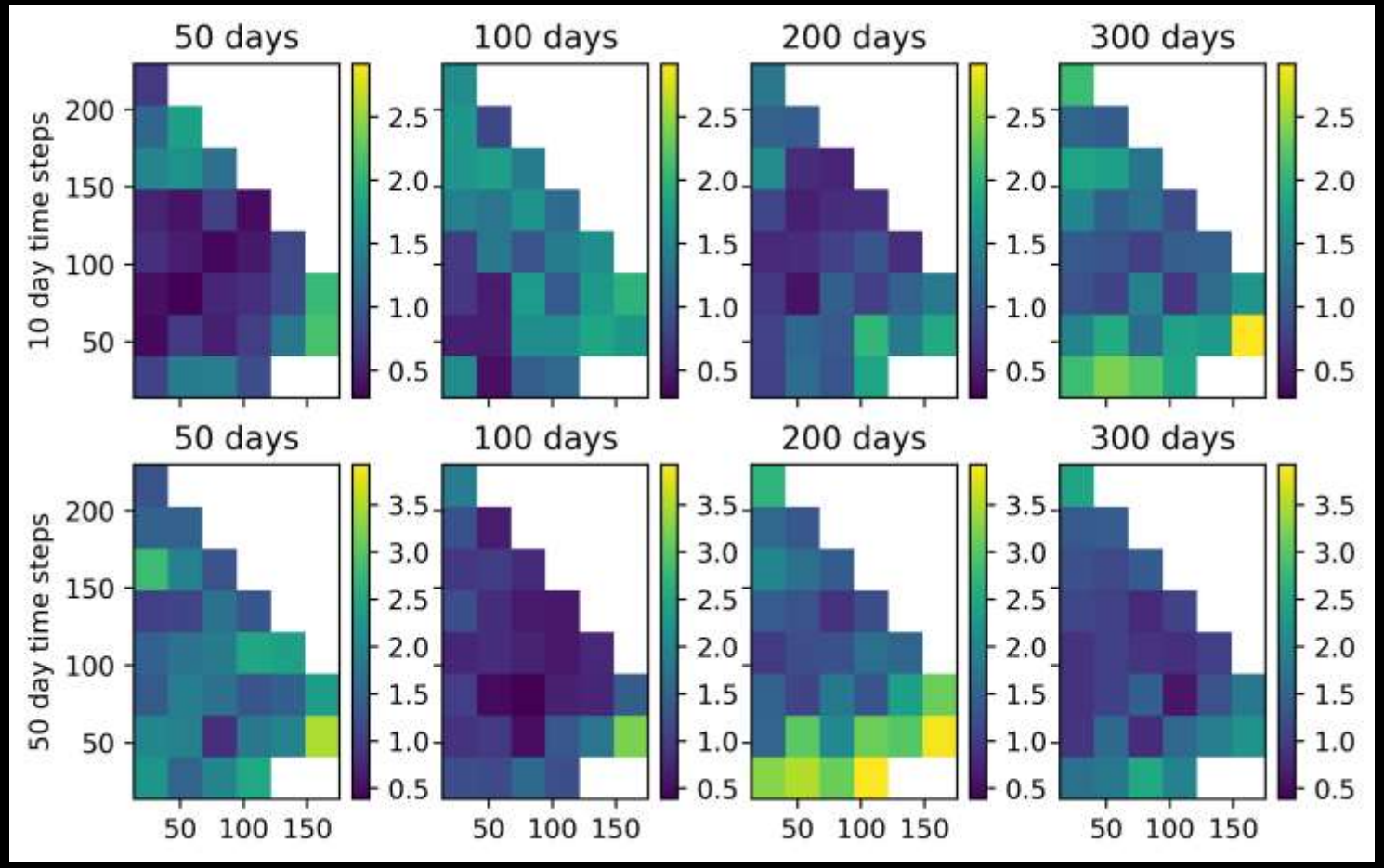
k_{eff} and $\delta\rho$ for full core analysis

FULL CORE – CUMULATIVE XENON



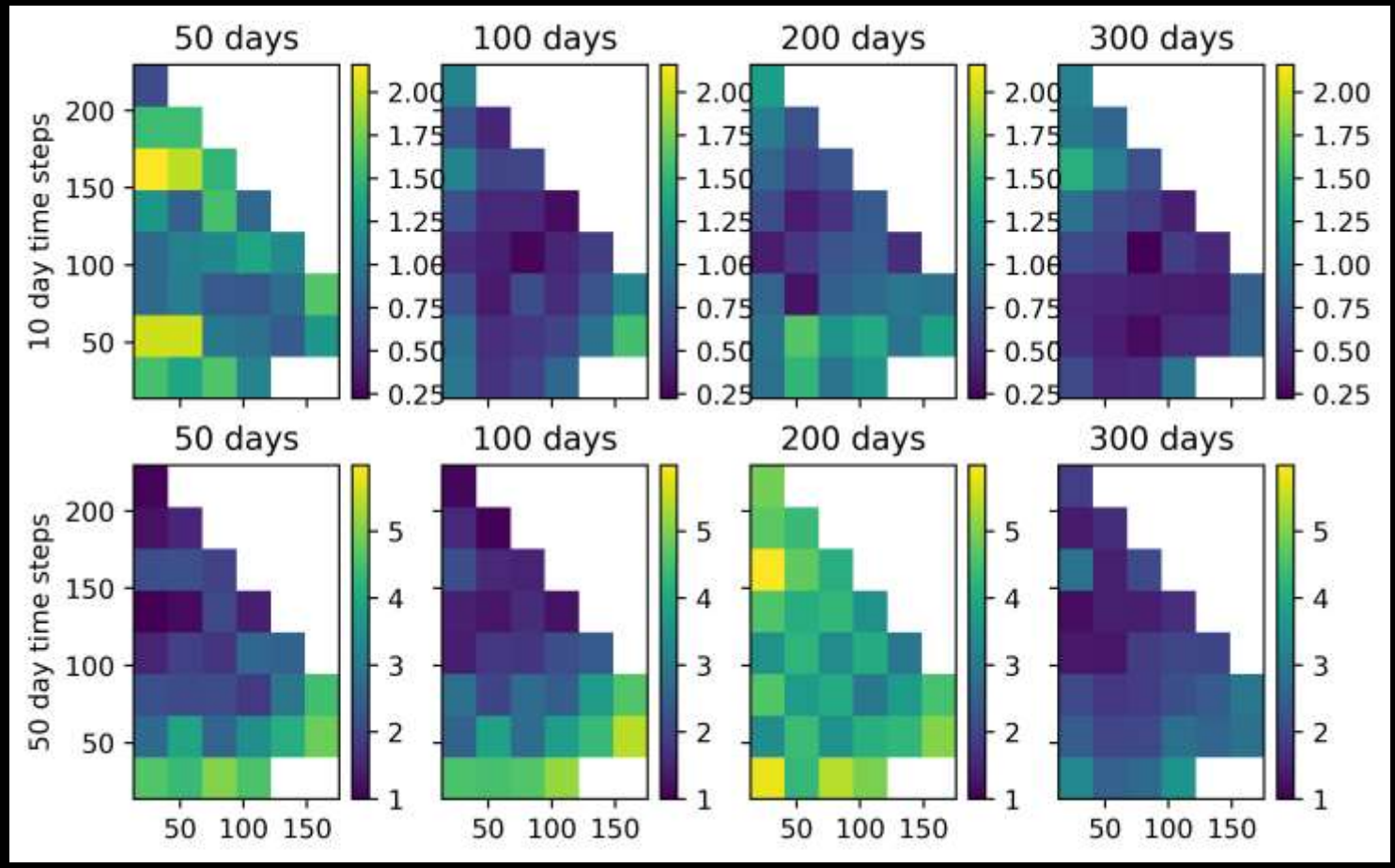
Absolute error in full core cumulative
Xe-135 density 50 day steps

FULL CORE – ASSEMBLY XENON ERRORS



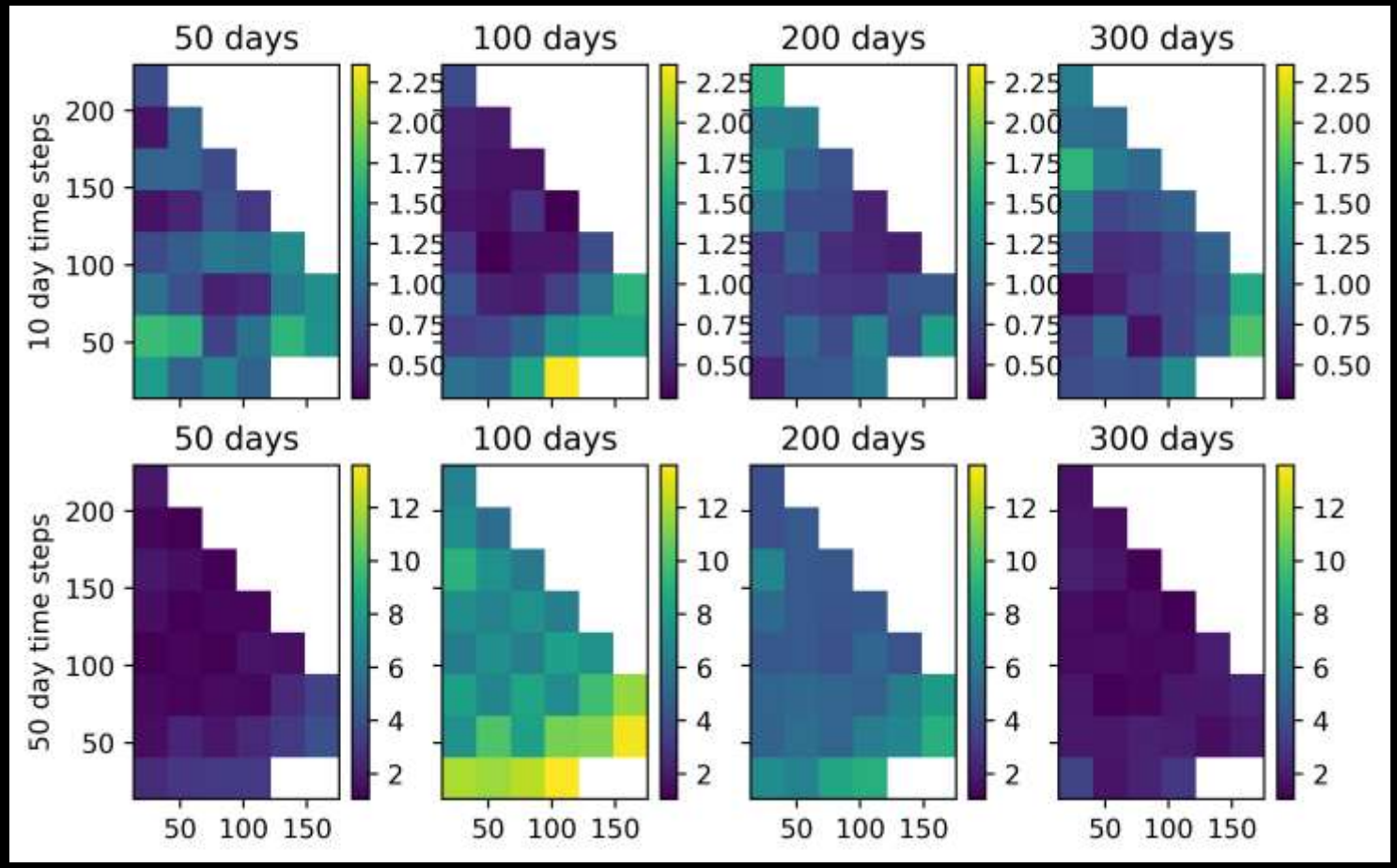
Maximum nodal xenon density error
using SIE method

FULL CORE – ASSEMBLY XENON ERRORS



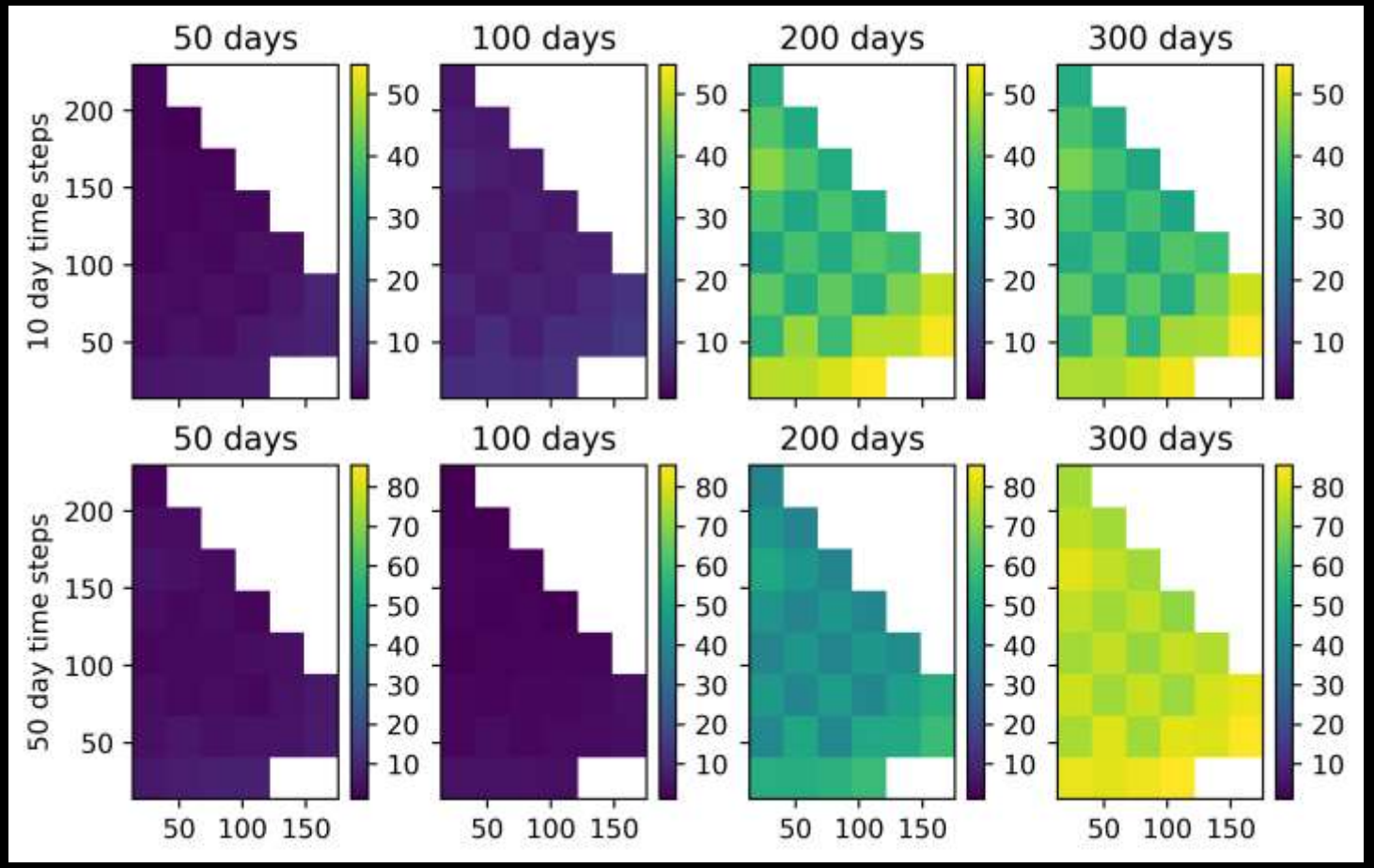
Maximum nodal xenon density error
using LE/QI method

FULL CORE – ASSEMBLY XENON ERRORS



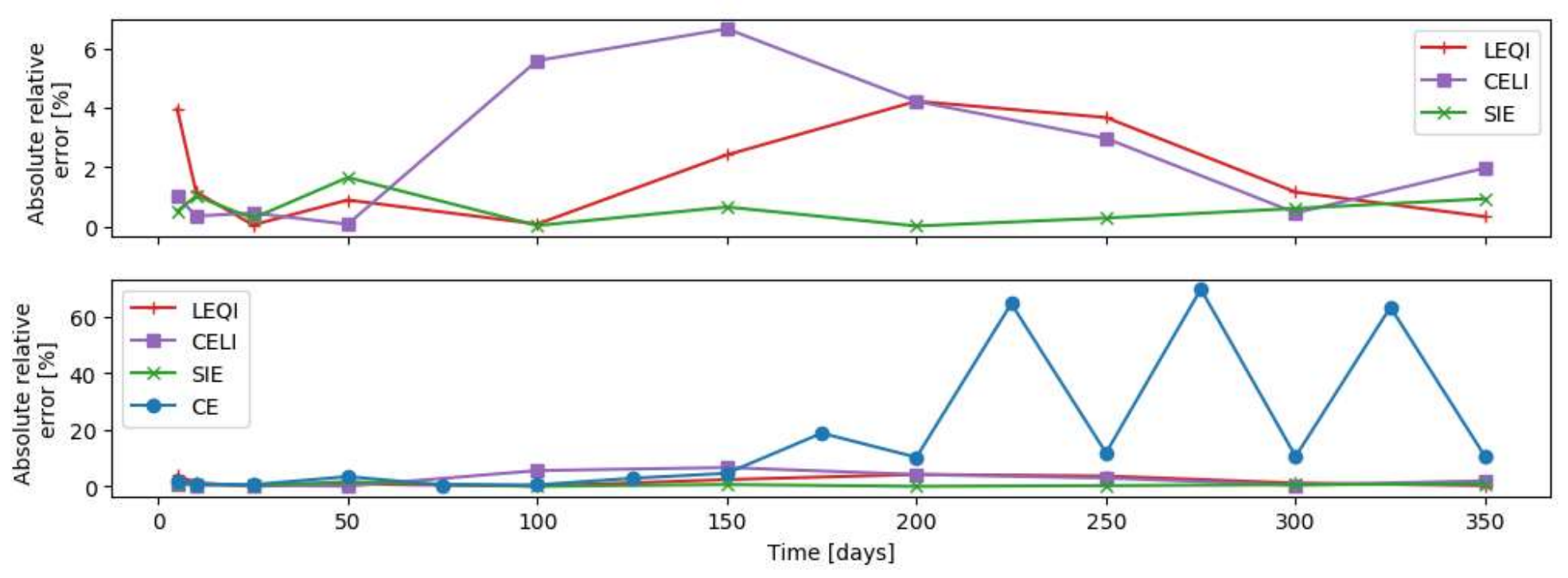
Maximum nodal xenon density error
using CE/LI method

FULL CORE – ASSEMBLY XENON ERRORS



Maximum nodal xenon density error
using CE method

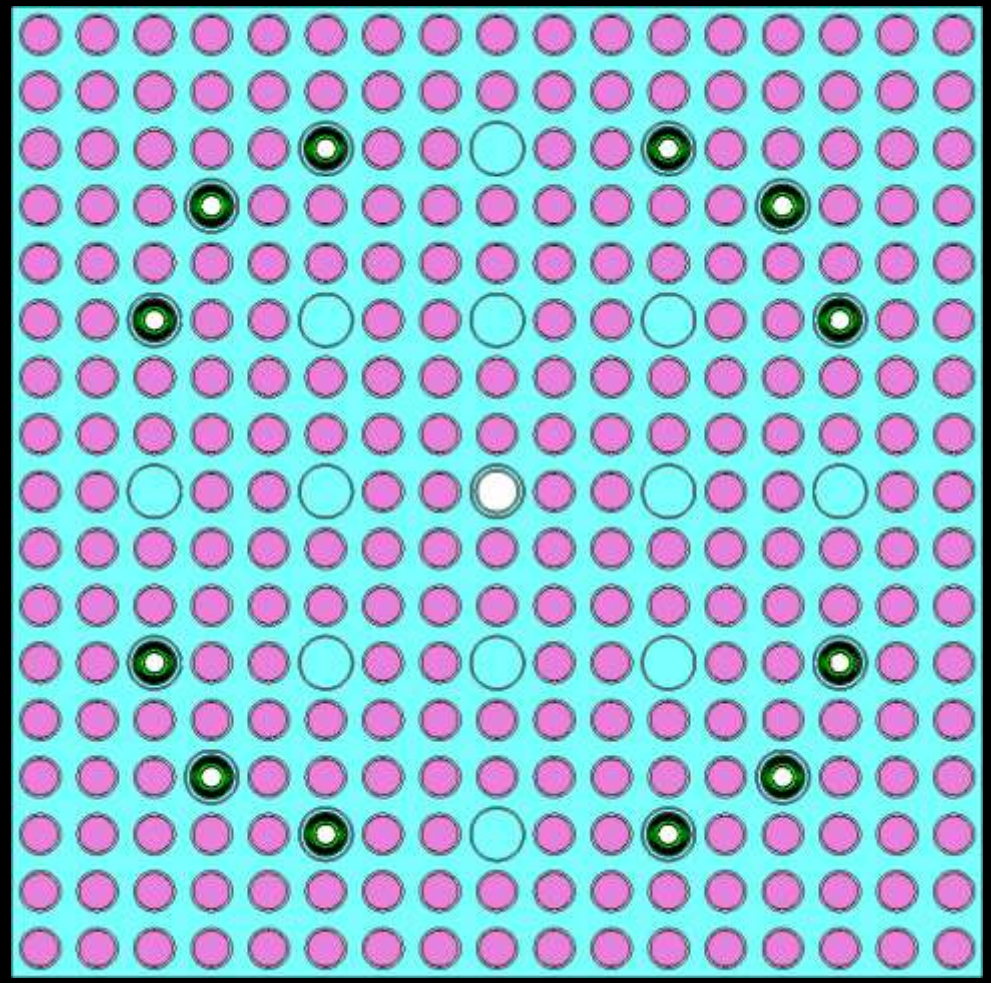
FULL CORE – NODAL XENON ERRORS



Xenon density and error for a single node – 50 day schedule

CONVERGENCE STUDY

- Further investigate the effect of decreasing time steps
- Single assembly, 12 burnable absorbers from reactor
- 16 axial layers
- 7500 particles/cycle
- 100/500 inactive/active cycles



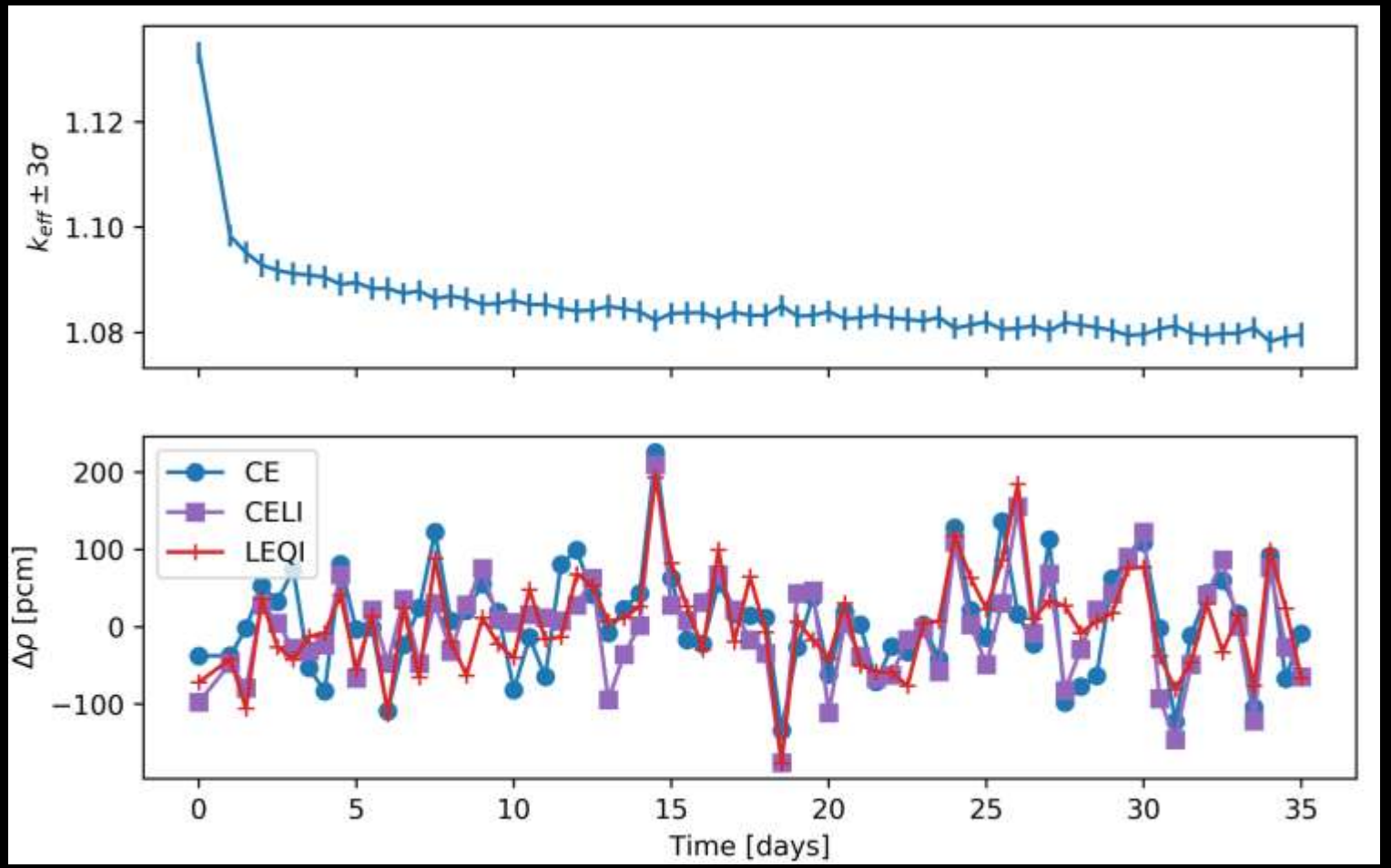
12BA assembly for convergence study

CONVERGENCE STUDY

- 35 day schedule
- 0.5 days * 10 for initial FP build up
- Remaining 30 days divided into integer number of equal-width intervals
- 30 days per step down to 0.25 days per step

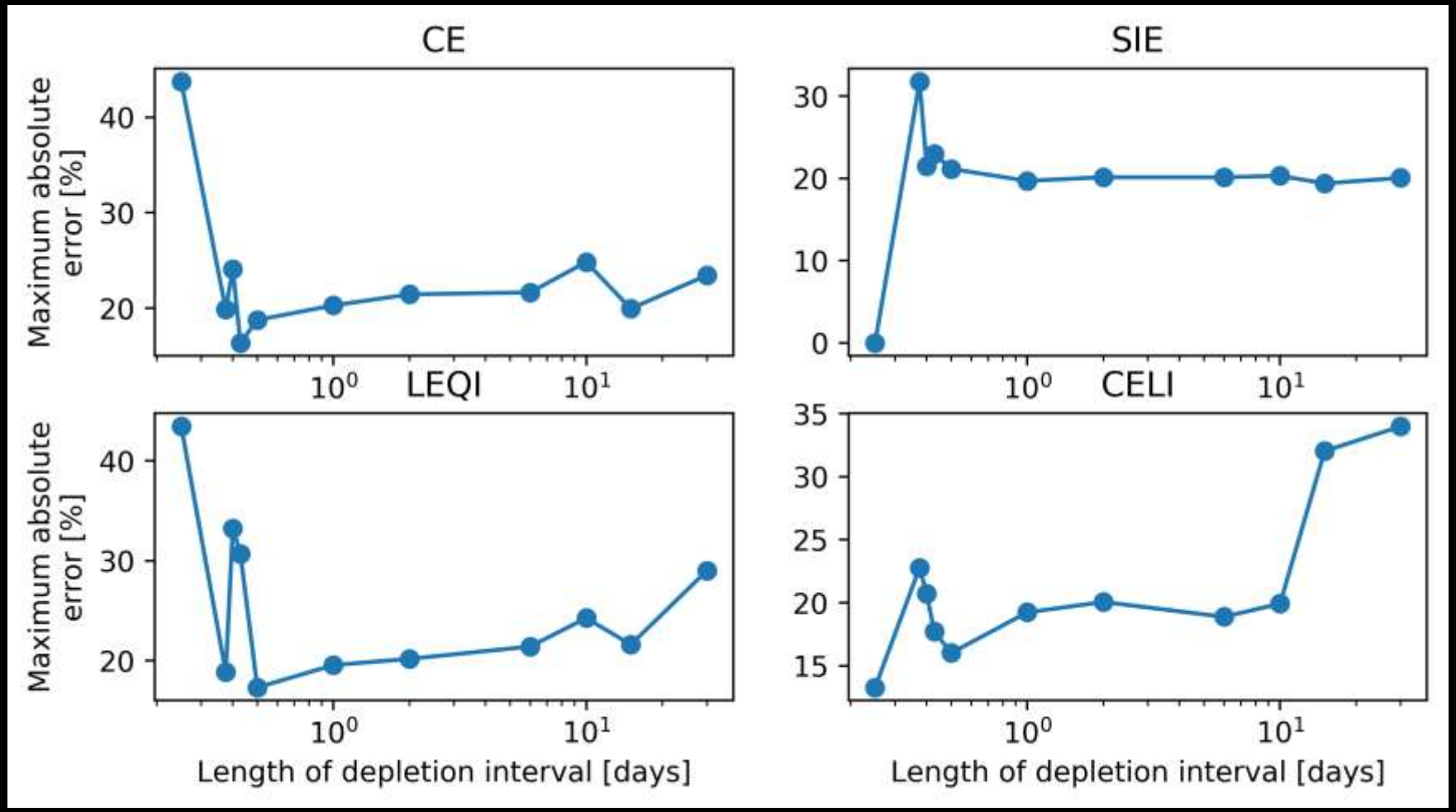
- 10 substeps per depletion interval – CE/LI and LE/QI
- 10 iterations per depletion interval – SIE
- Reference – 0.25 days per depletion interval

CONVERGENCE STUDY



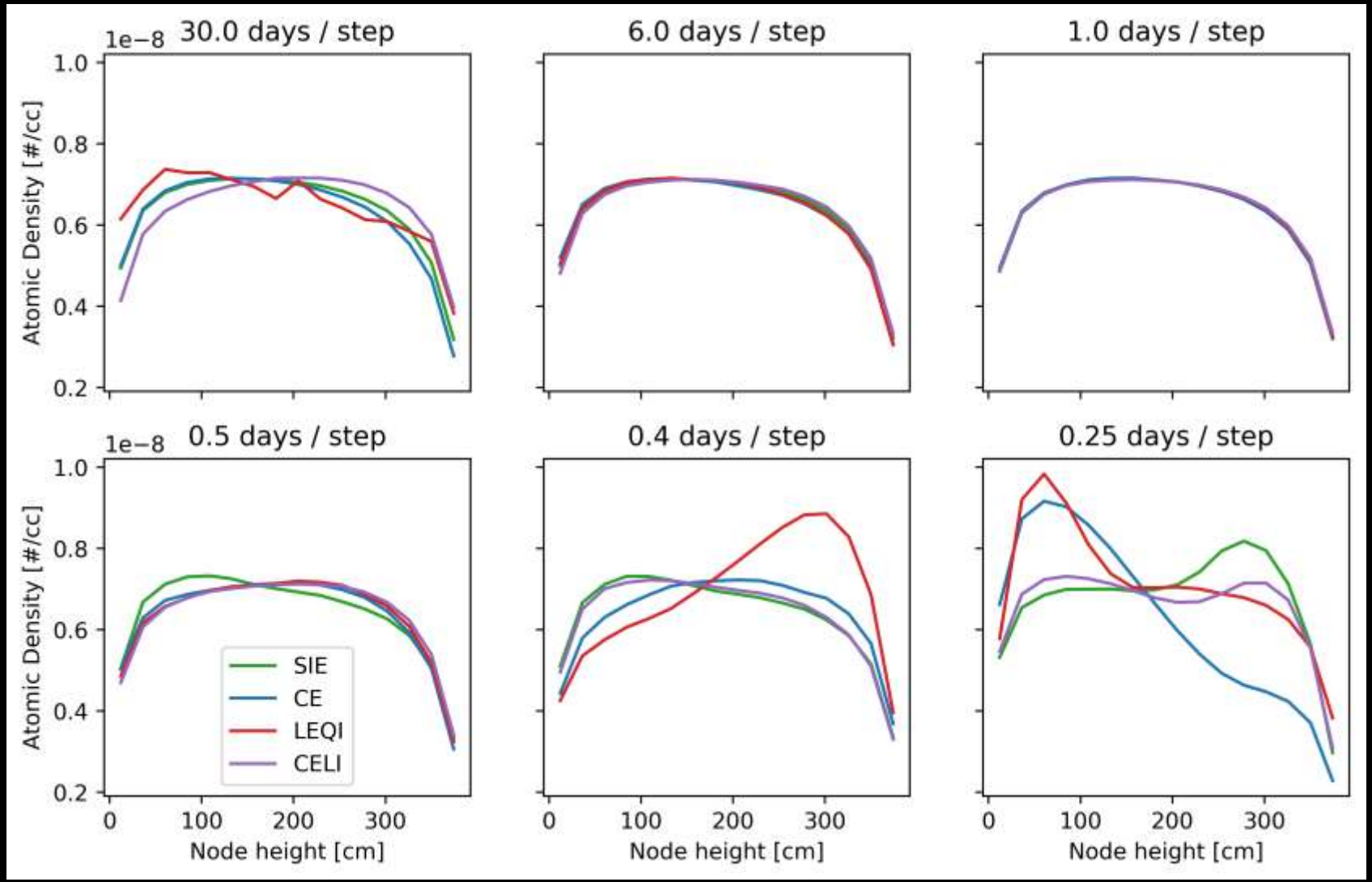
k_{eff} and $\delta\rho$ for methods with 0.5 day steps

ASSEMBLY– XENON ERROR CONVERGENCE



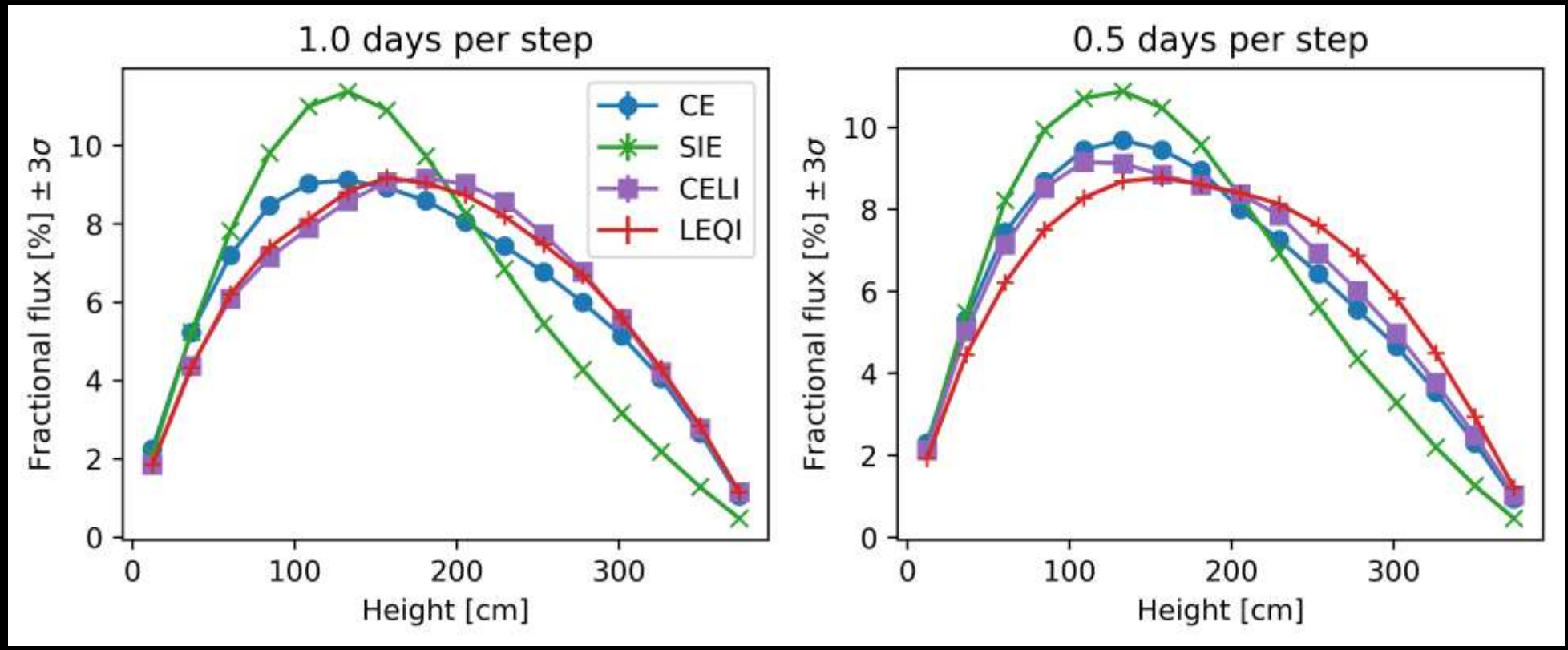
Maximum nodal xenon density error at EOL vs.
length of depletion interval

ASSEMBLY – EOL XENON



Xenon distribution at end of cycle for various depletion interval lengths

ASSEMBLY – NODAL FLUX SHARE



Nodal flux share at EOL for 1 and 0.5 day steps for all methods

CONCLUSION AND DISCUSSION

Full Core Analysis

- ≥500 pcm error in k_{eff}
- 20% error in local xenon density
  - Similar results were obtained for other fission products
- Methods appeared to converge towards the reference solution at EOL

Assembly Analysis

- Some indication of error convergence for reducing time steps
- Divergence for time steps below one day
- 10 % deviation in nodal flux

CONCLUSION AND DISCUSSION

- As with this work, reference cases are often those with the smallest time step
- Trend towards transient and life-cycle analysis with Monte Carlo methods
- Accuracy of depletion method will drive accuracy of transport solution

- Further work on improved coupling schemes
- Perturbation theory-based methods
- More points for extrapolation/interpolation
- Benchmarking

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THANK YOU

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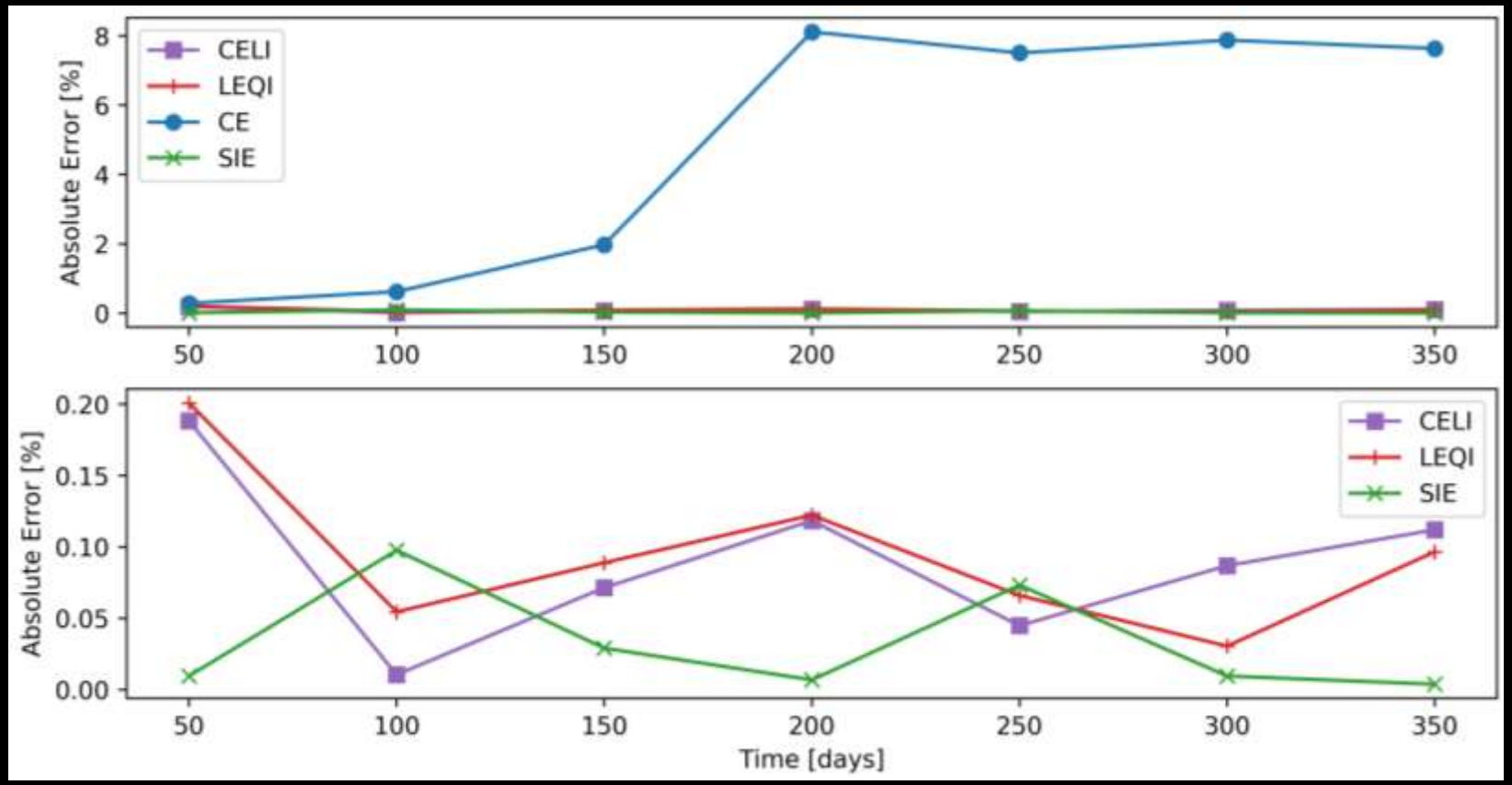
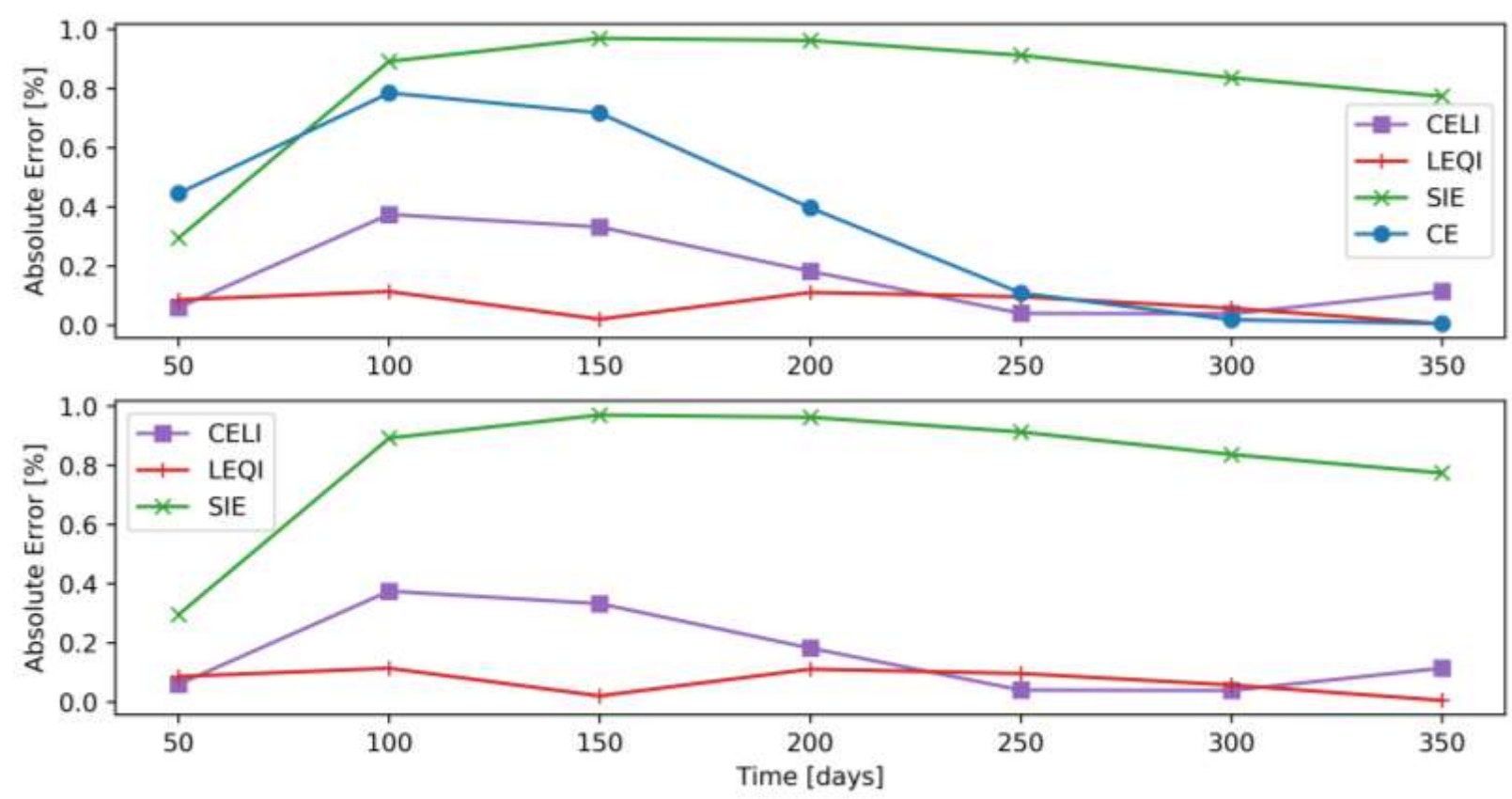
FULL CORE - XENON



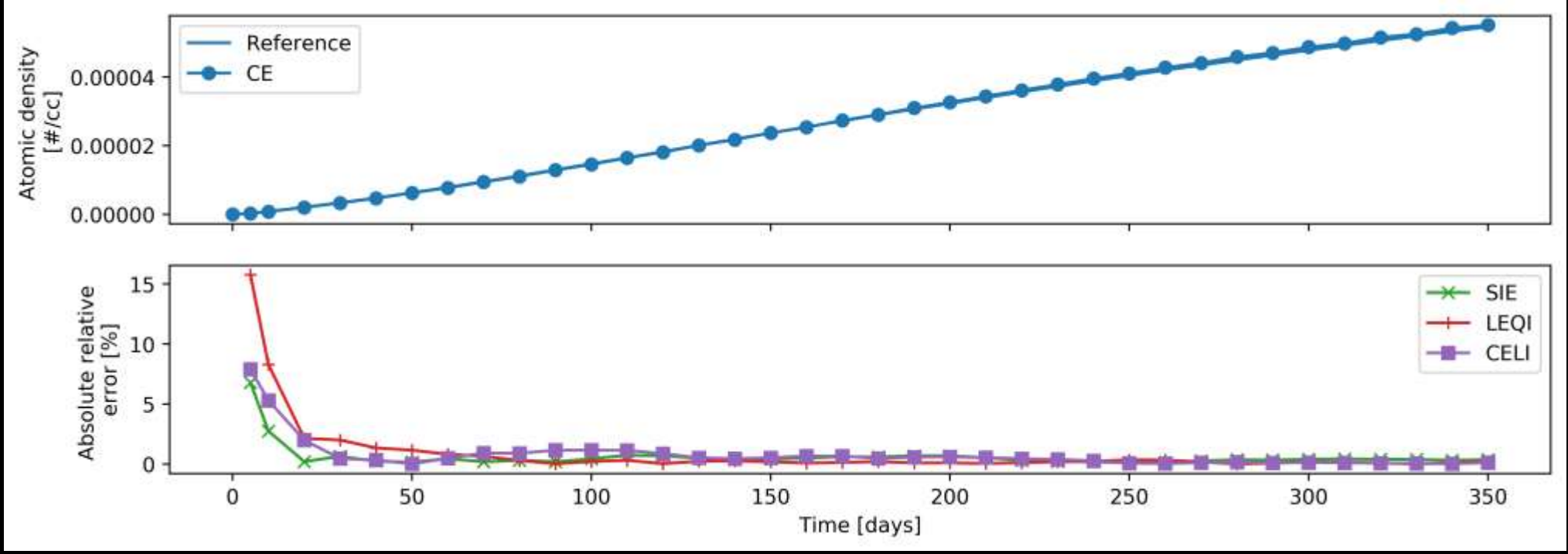
Error in cumulative Xenon density for full core.
10 day steps, plotted at 50 day increments

FULL CORE – PU239



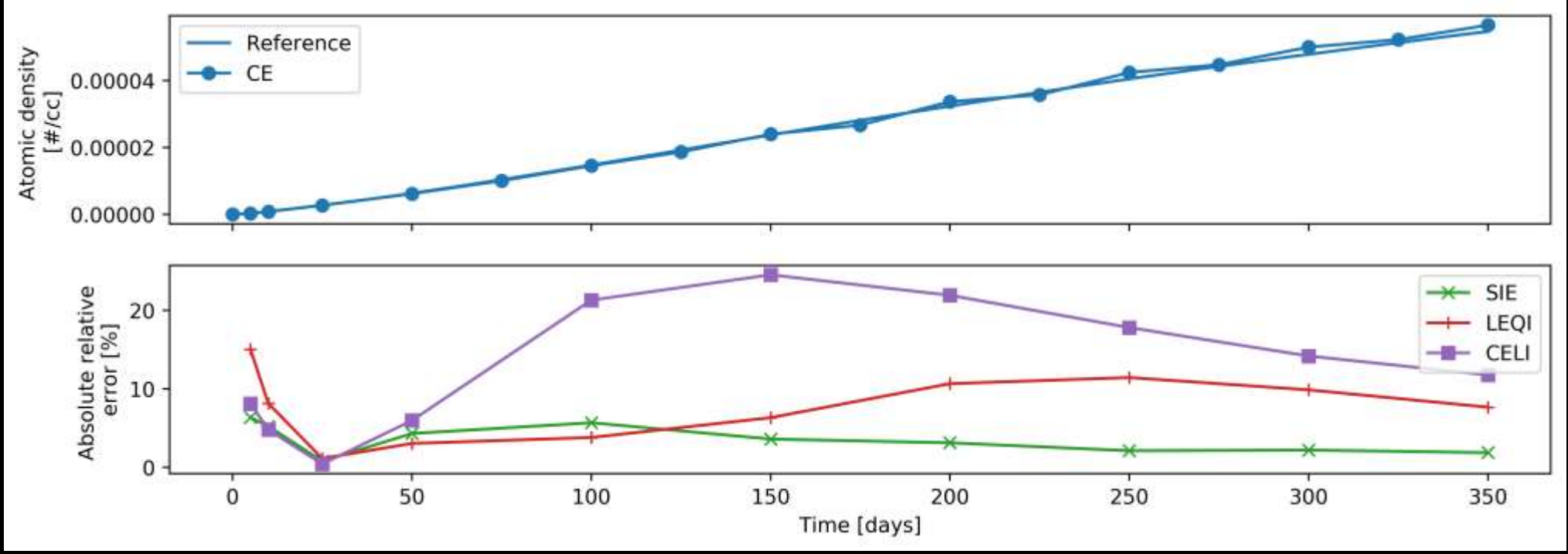
Cumulative error in Pu-239 density - 50 day steps

FULL CORE – NODAL PU239



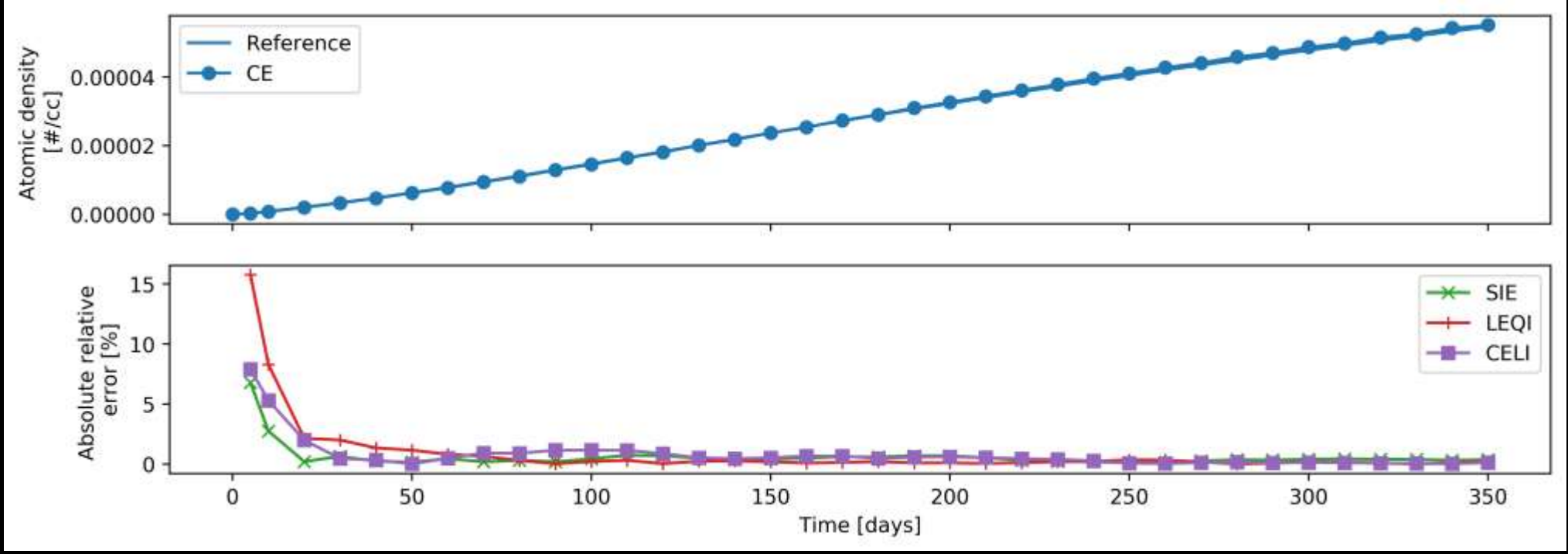
Absolute error in nodal Pu-239 using 10 day time steps

FULL CORE – NODAL PU239



Absolute error in nodal Pu-239 using 50 day time steps

FULL CORE – NODAL PU239



Absolute error in nodal Pu-239 density with 10 day steps