

# Perturbation-based coupling of Monte Carlo and Burn-up for multiple burnable regions

Paul Cosgrove, Eugene Shwageraus

08/11/17

Engineering - Energy, Fluid dynamics and Turbo-machinery

# Contents

- Introduction: Burn-up Methods
- Introduction: Collision History Method
- Previous Work
- Extension to multiple burnable regions
- Implementation of perturbation-based depletion
- Results: Accuracy & Stability

# Burn-up

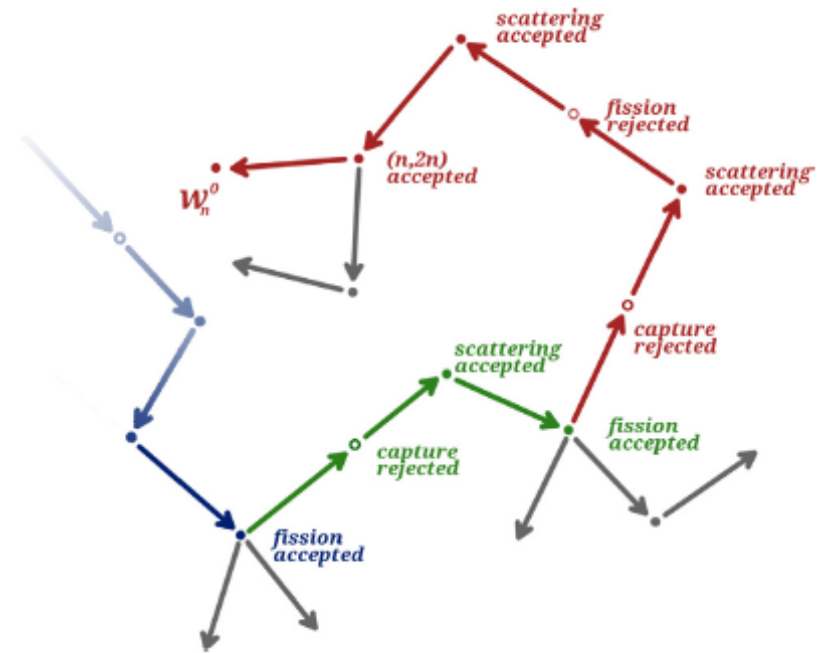
- Time evolution of fuel composition in a reactor
- Euler, Predictor-Corrector, Substep, Stochastic Implicit Euler (SIE)...

$$\frac{d\vec{N}}{dt} = \bar{\bar{\mathbf{A}}}(\sigma, \phi)\vec{N}(t)$$

- Different strengths and weaknesses...
- Want a stable and accurate method with minimum expense

# Sensitivity Calculations in Serpent

- Collision history method
- Can be used to generate  $\frac{\partial \sigma_i}{\partial \Sigma_{t,j}}$



$$w_n^* \simeq w_n^0 \cdot \left(1 + \frac{d\Sigma_{n,2n}}{\Sigma_{n,2n}}\right) \cdot \left(1 + \frac{d\Sigma_s}{\Sigma_s}\right) \cdot \left(1 - \frac{d\Sigma_f}{\Sigma_f}\right) \cdot \left(1 + \frac{d\Sigma_s}{\Sigma_s}\right) \cdot \left(1 - \frac{d\Sigma_c}{\Sigma_c}\right) \cdot \left(1 + \frac{d\Sigma_f}{\Sigma_f}\right) \cdot \left(1 + \frac{d\Sigma_s}{\Sigma_s}\right) \cdot \left(1 - \frac{d\Sigma_c}{\Sigma_c}\right) \cdot \left(1 + \frac{d\Sigma_f}{\Sigma_f}\right) \dots$$

# Perturbation-based Depletion: Previous work

$$S_x^R = \frac{\partial R / R}{\partial x / x} = \frac{\partial \sigma_i}{\partial \Sigma_{t,j}} * \frac{\Sigma_{t,j,0}}{\sigma_{i,0}} = \frac{\partial \sigma_i}{\partial N_j} * \frac{N_{j,0}}{\sigma_{i,0}}$$

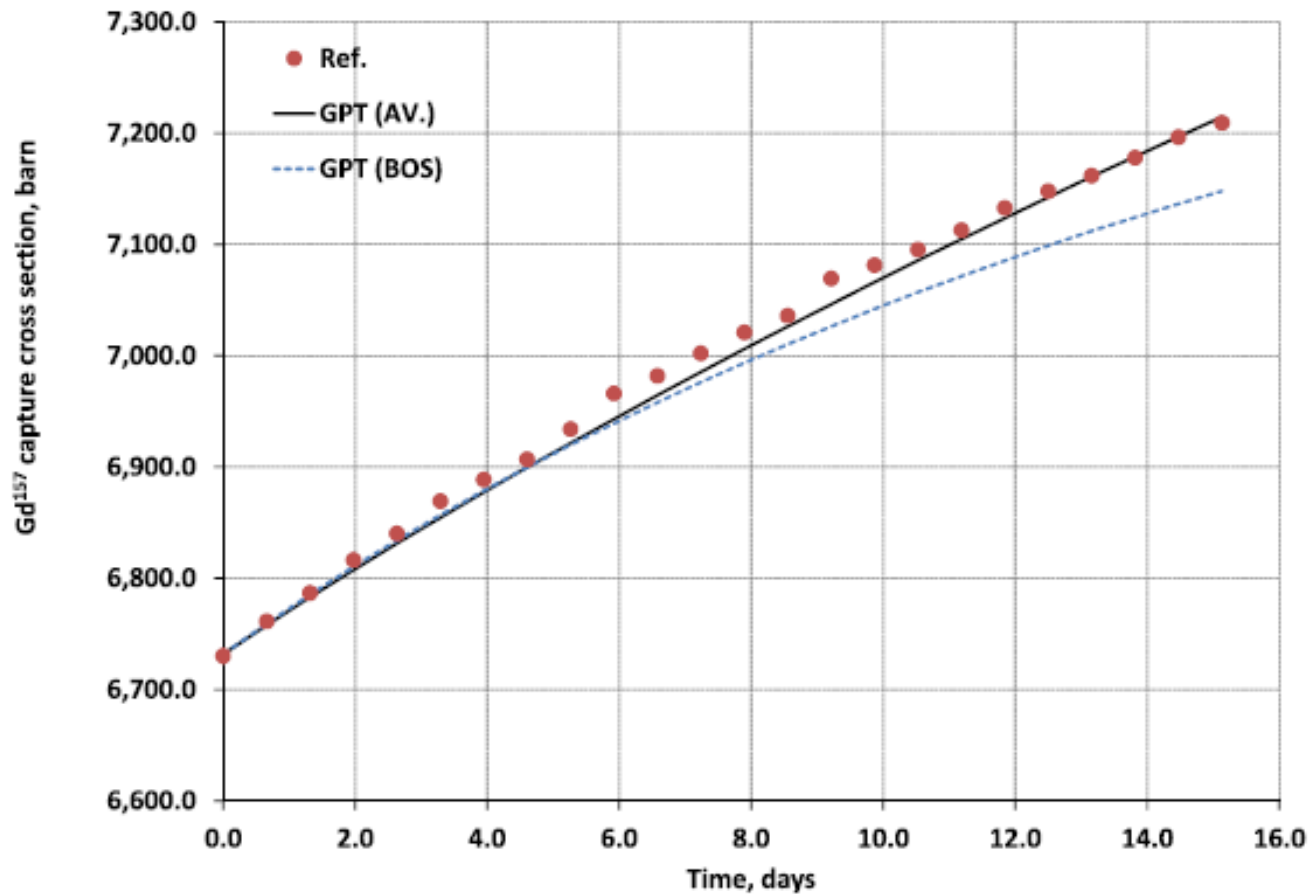
Can calculate partial derivatives of 1-group XS's to any nuclide density

# Perturbation-based Depletion: Previous work

$$\sigma_x(\vec{N}) = \sigma_x(\vec{N}_0) + \frac{\partial \sigma_x}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_0]$$

Can create low-order approximation for how XS's change with burn-up

# Time (or burn-up) dependence of Gd-157 XS



# Extension to multiple burnable regions

- Single region was proof of principle
- Depletion problems are necessarily multi-region – Gadolinium pins require fine spatial discretisation for physically accurate analysis
- As well as accuracy in nuclide density, multiple regions raise concerns over spatial stability



# Extension to multiple burnable regions

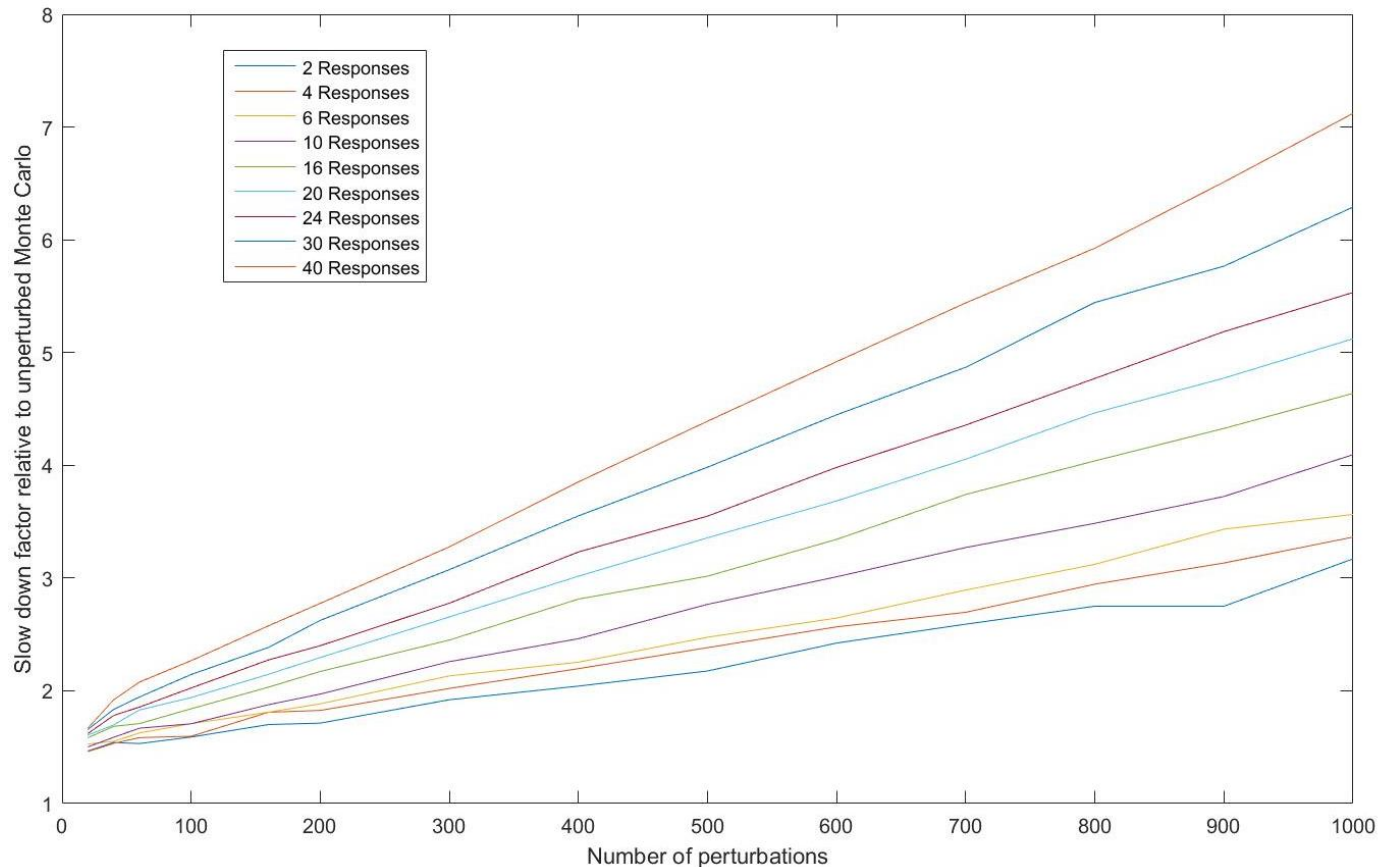
- XS's in one region now sensitive to perturbations in all other regions
- Requires an additional flux dependence

$$\phi(\vec{N}) = \phi(\vec{N}_0) + \frac{\partial \phi}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_0]$$

- Serpent 2.1.29 more efficiently tallies large numbers of sensitivities across multiple regions – previously impossible

# Implementation of burn-up scheme

Previously took sensitivity to every perturbation – no longer practical!



# Implementation of burn-up scheme

- Need some degree of adaptivity – choose which responses and perturbations are worth including!
- Also must account for uncertainty: zero any  $S^R_x$  with uncertainty  $> 10\%$

# Implementation of burn-up scheme

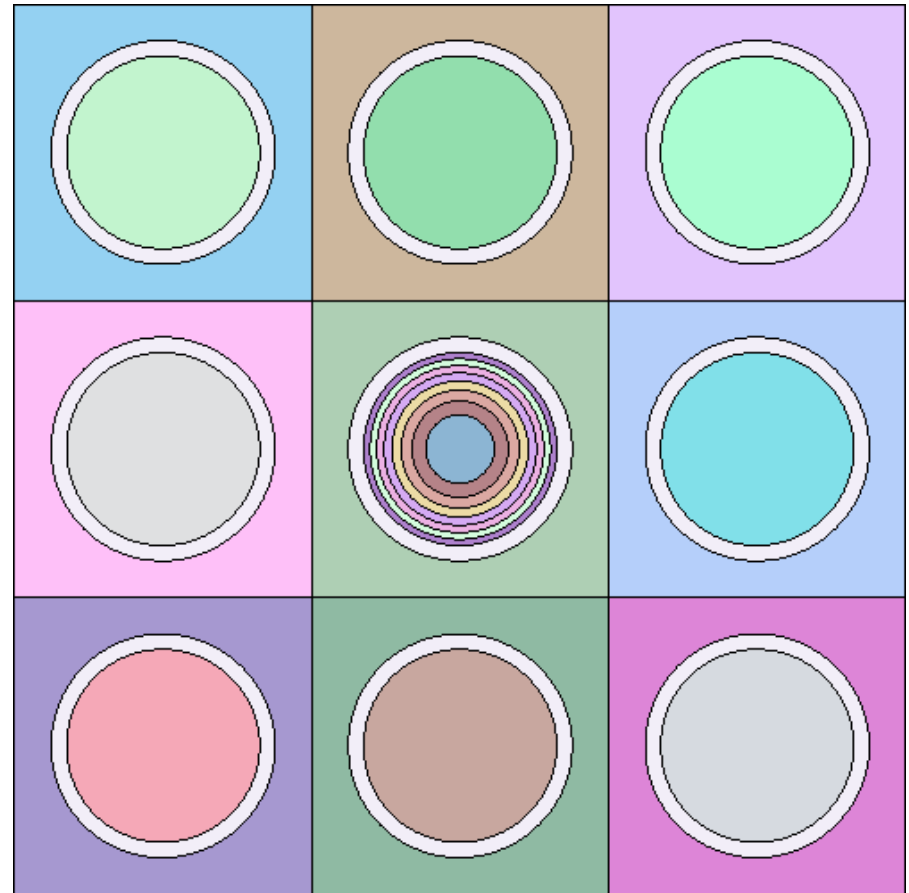
- Modify Serpent to extract burn-up information for external script
- Requires 2 transport solutions per BOS/EOS (Serpent quirk)
- Substep procedure: LE/QI or LE/LI with perturbations

$$\begin{aligned} \sigma_x(t) &= \frac{t - t_1}{t_0 - t_1} \sigma_x(t_0) \left( 1 + \frac{\partial \sigma_x(t_0)}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_0] \right) \\ &+ \frac{t - t_0}{t_1 - t_0} \sigma_x(t_1) \left( 1 + \frac{\partial \sigma_x(t_1)}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_1] \right) \end{aligned}$$

- Regular substep for XS's which do not have sensitivity coefficients

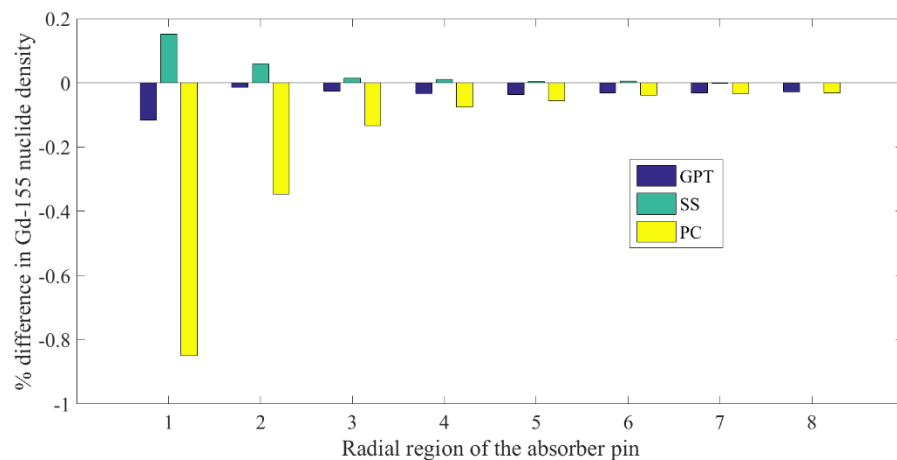
# Results: Mini-assembly

- 2D, 3x3 mini-assembly with reflective boundaries
- Contains radially divided Gd-bearing pin
- 2.5% Gd-155 by nuclide density
- Burned for 100 days by Pred-Corr, Substep and perturbation scheme
- Reference LE/QI solution

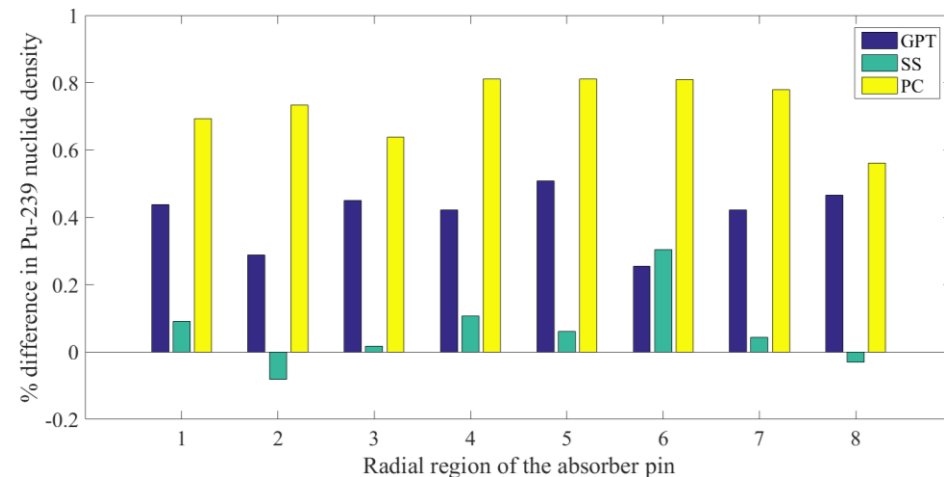


# Results: Mini-assembly

- Good accuracy in predicting Gd-155
- Not so good in other elements – outclassed by Substep
- Runtime a factor of 17x slower on final burn-up step (neglecting cost of second MC solution)



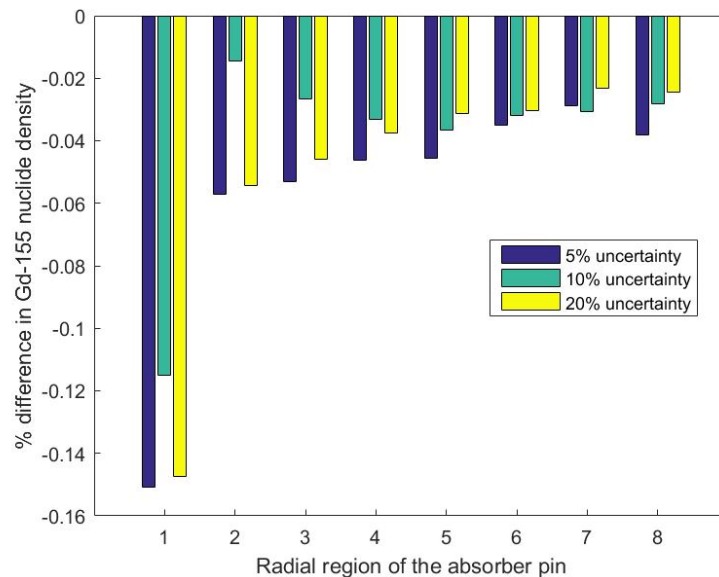
Gd-155



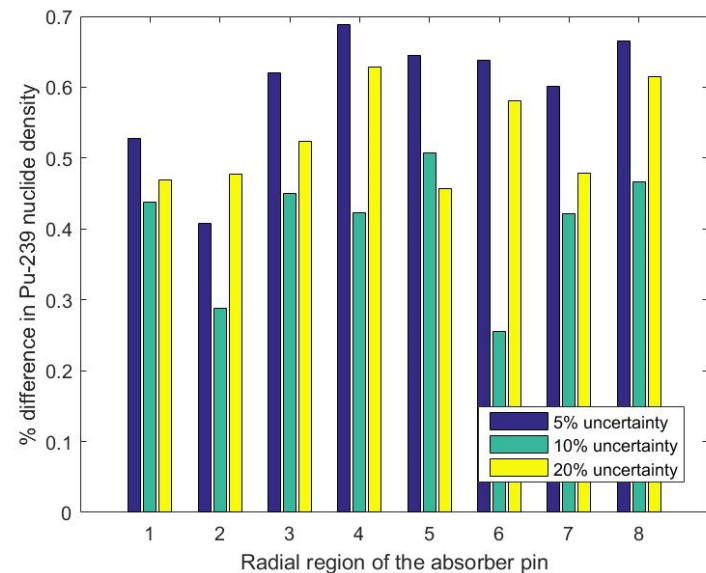
Pu-239

# Results: Mini-assembly

- Briefly examined the effect of varying the uncertainty threshold from 5% to 20%
- 10% appears to be roughly optimal for agreement with benchmark



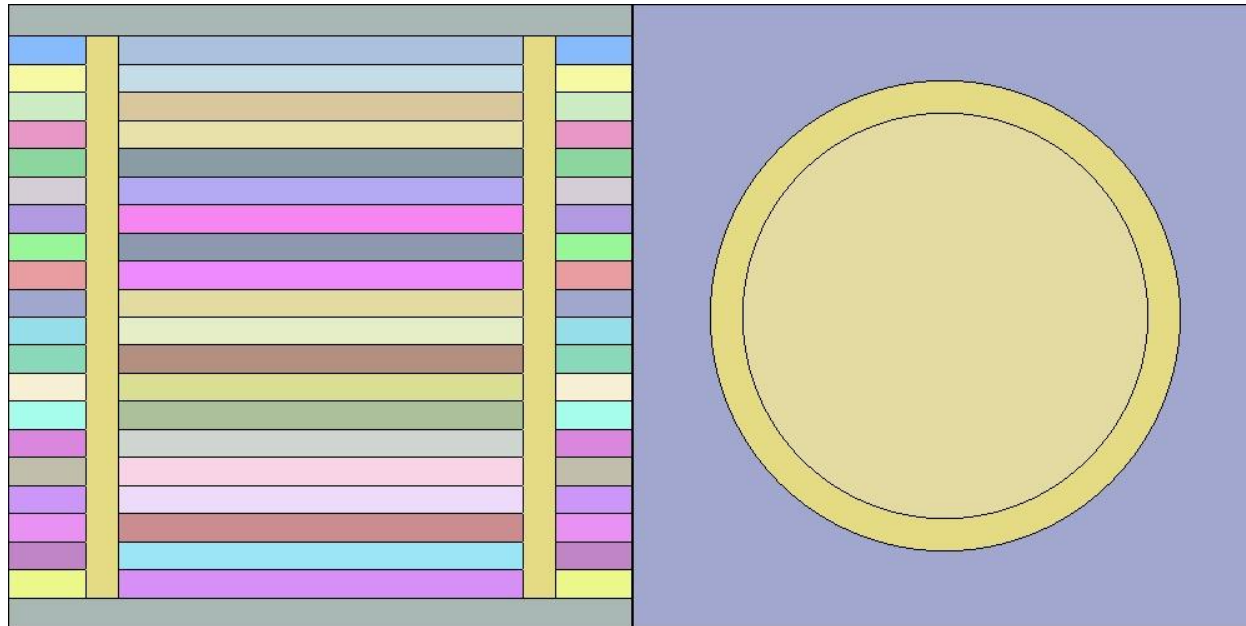
Gd-155



Pu-239

# Results: PWR pin

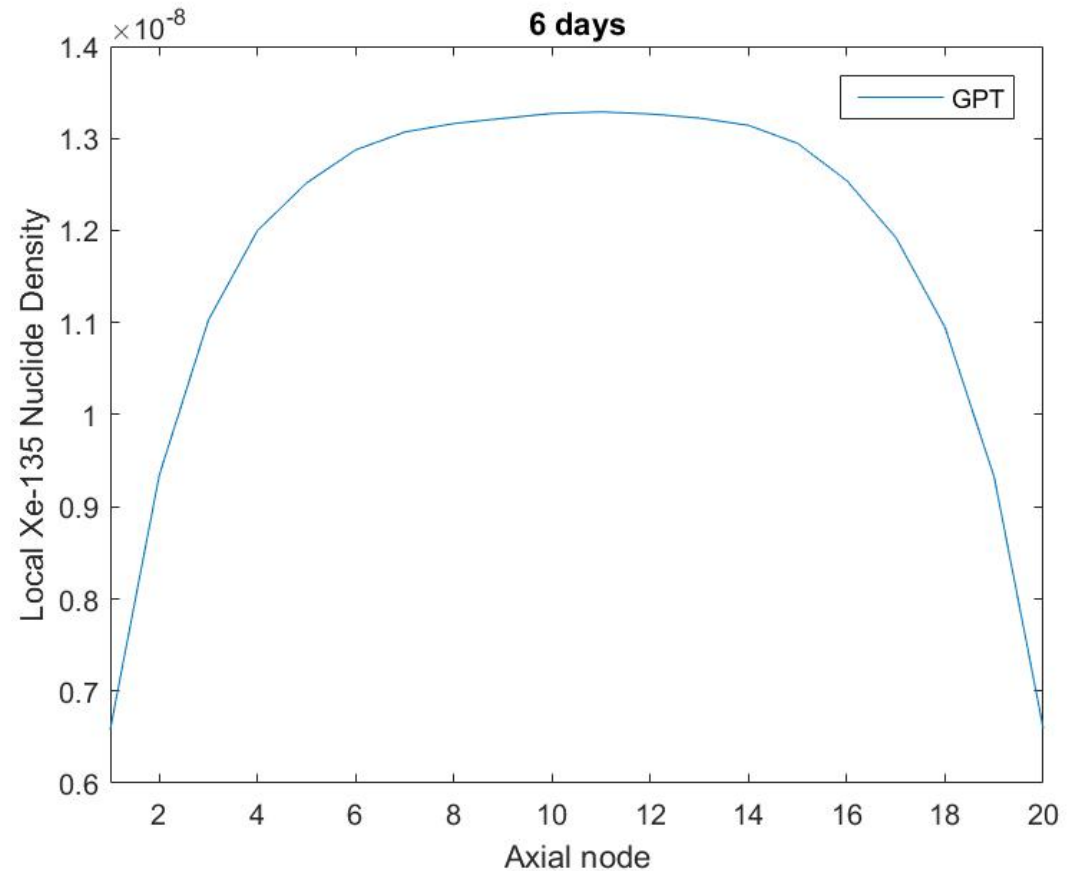
- 3D, 3.66m tall PWR pin with radially reflective and axially vacuum BCs
- Divided into 20 regions with identical properties and moderator density
- Examine spatial stability with burnup – burn for 310 days with 20 day steps





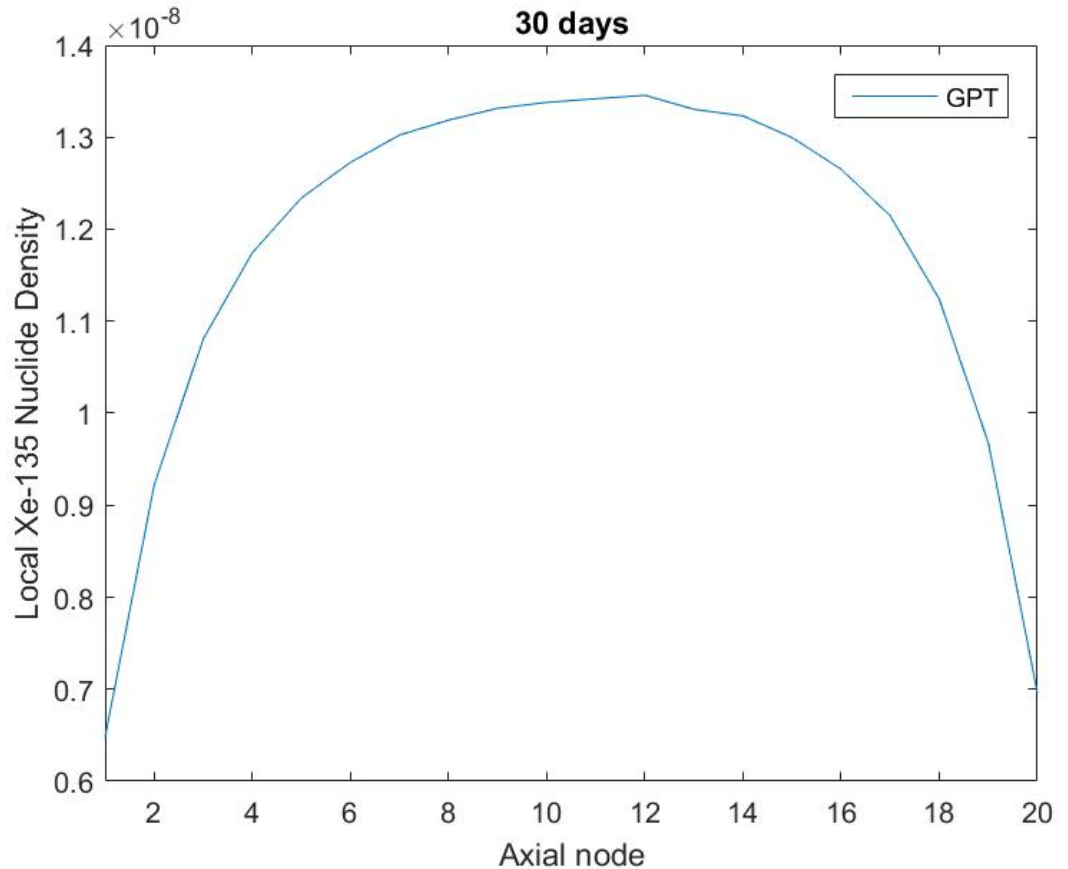
# Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability



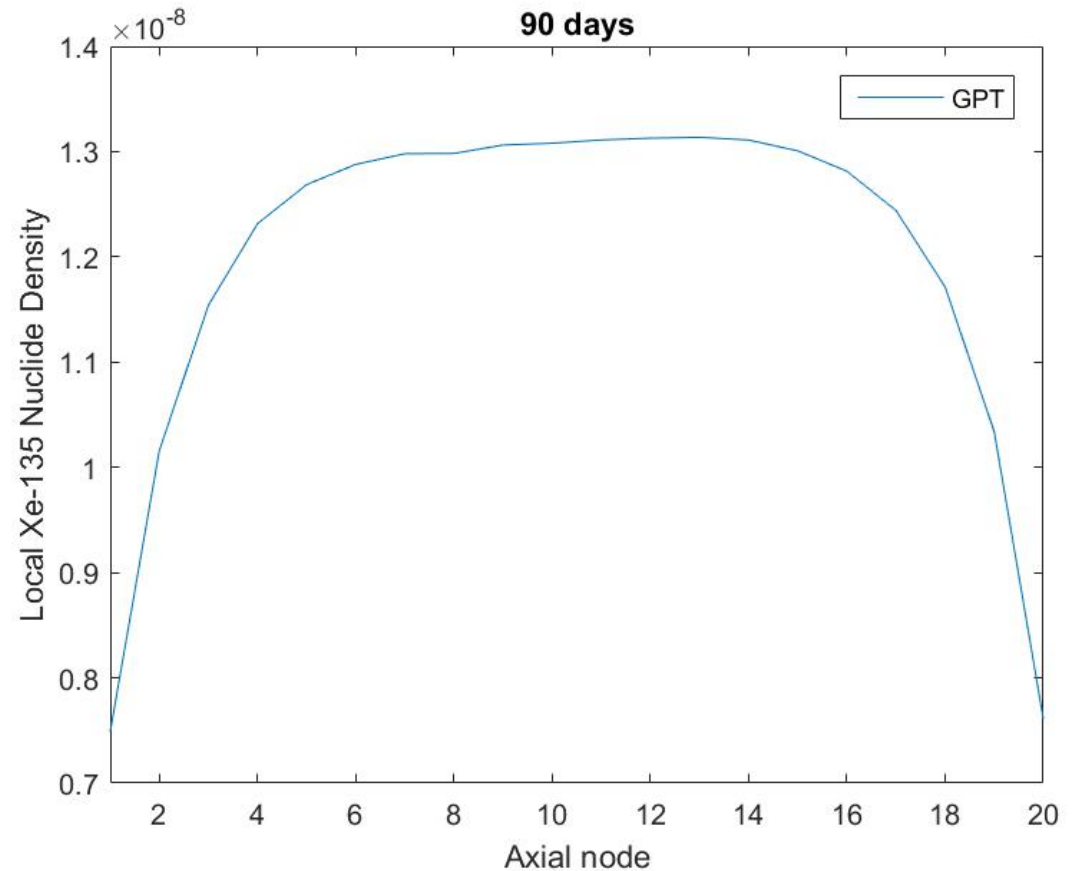
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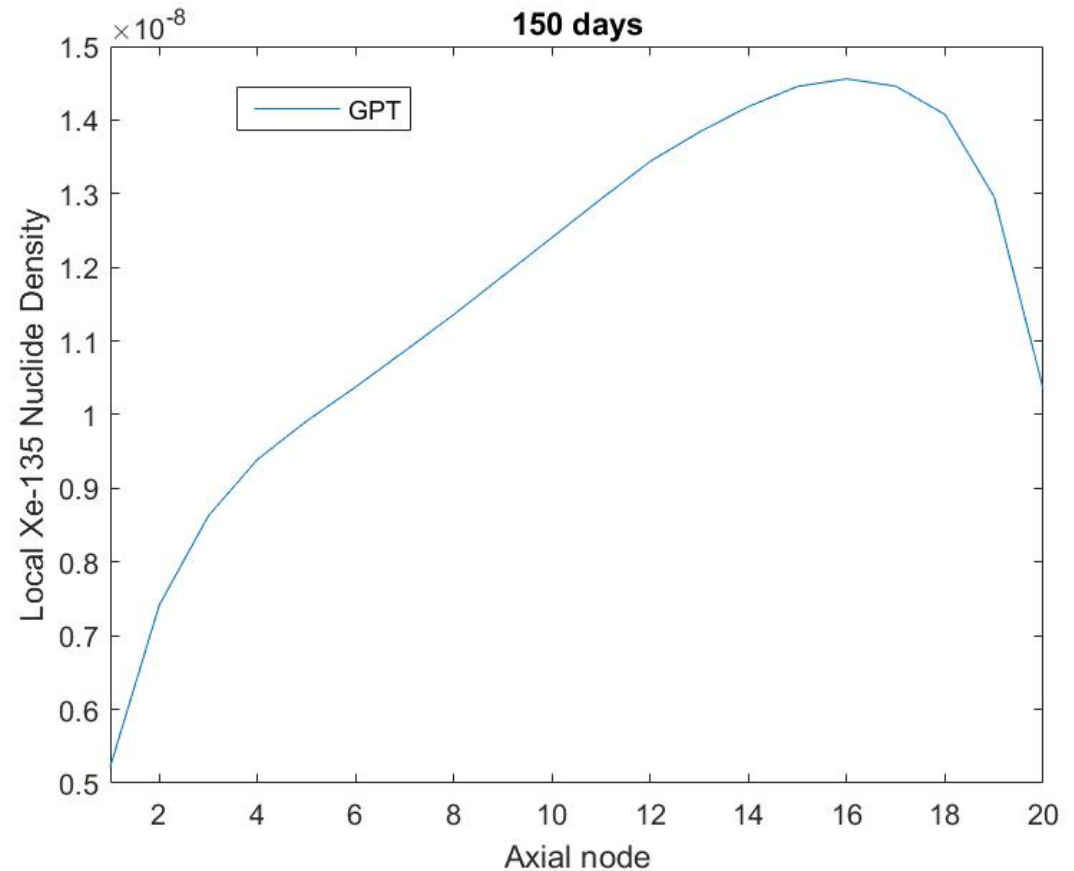
# Results: PWR pin

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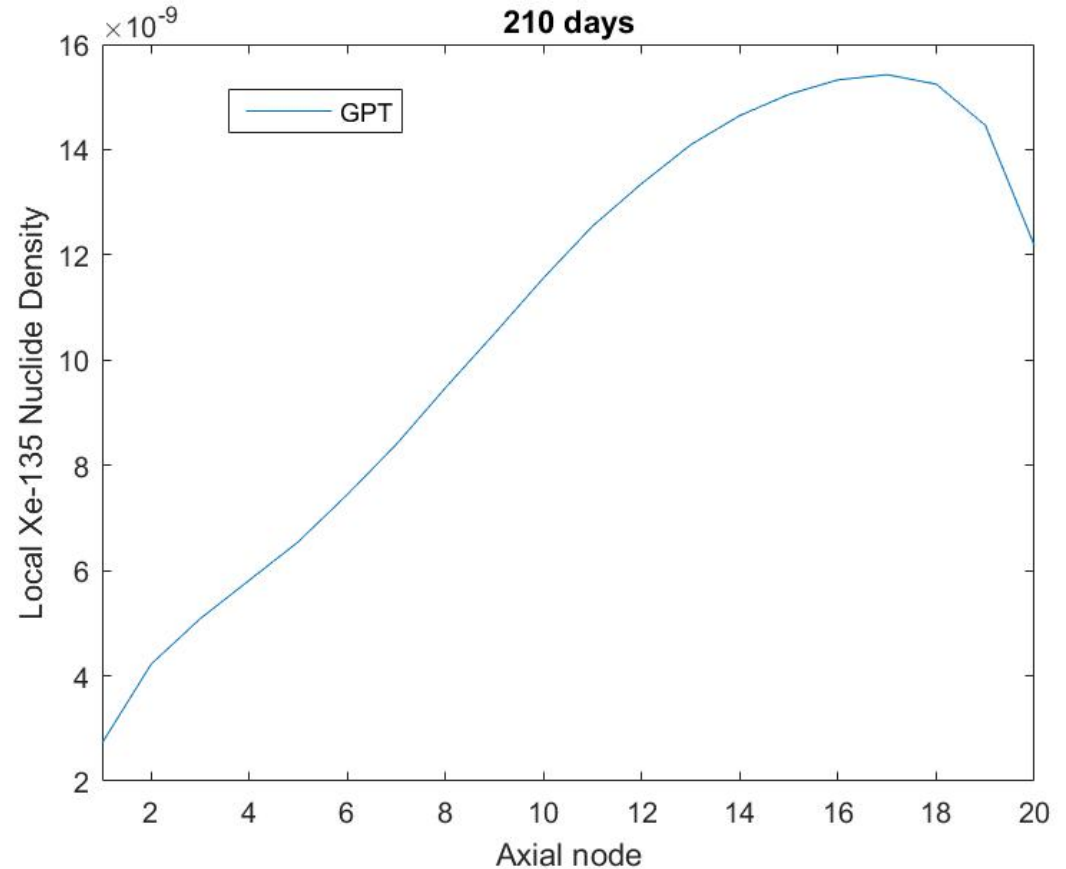
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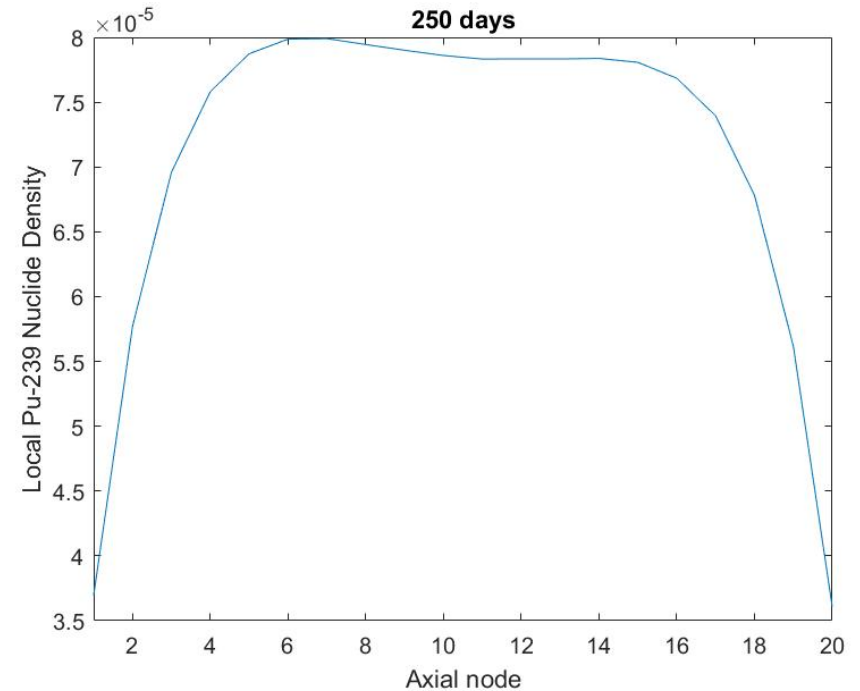
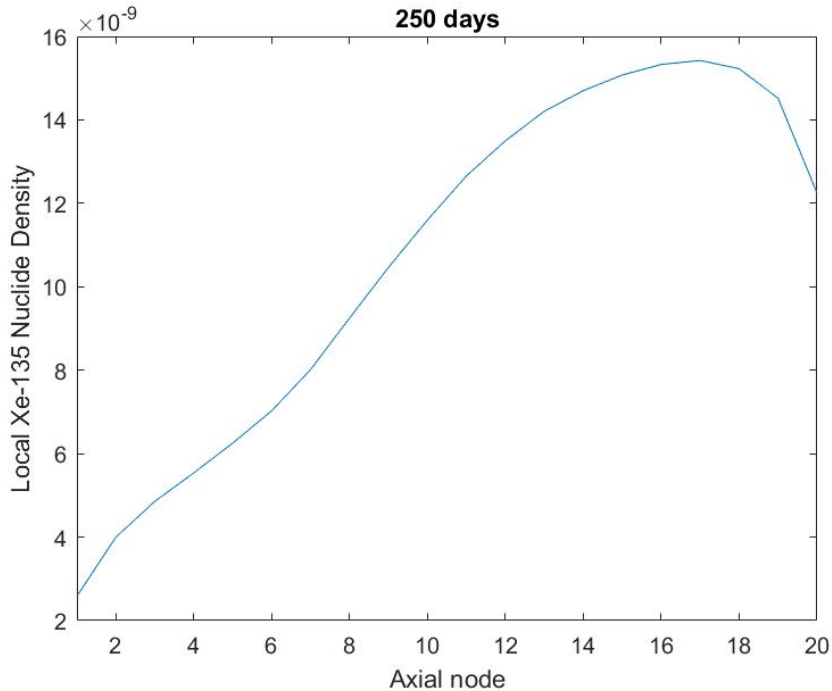


# Results: PWR pin

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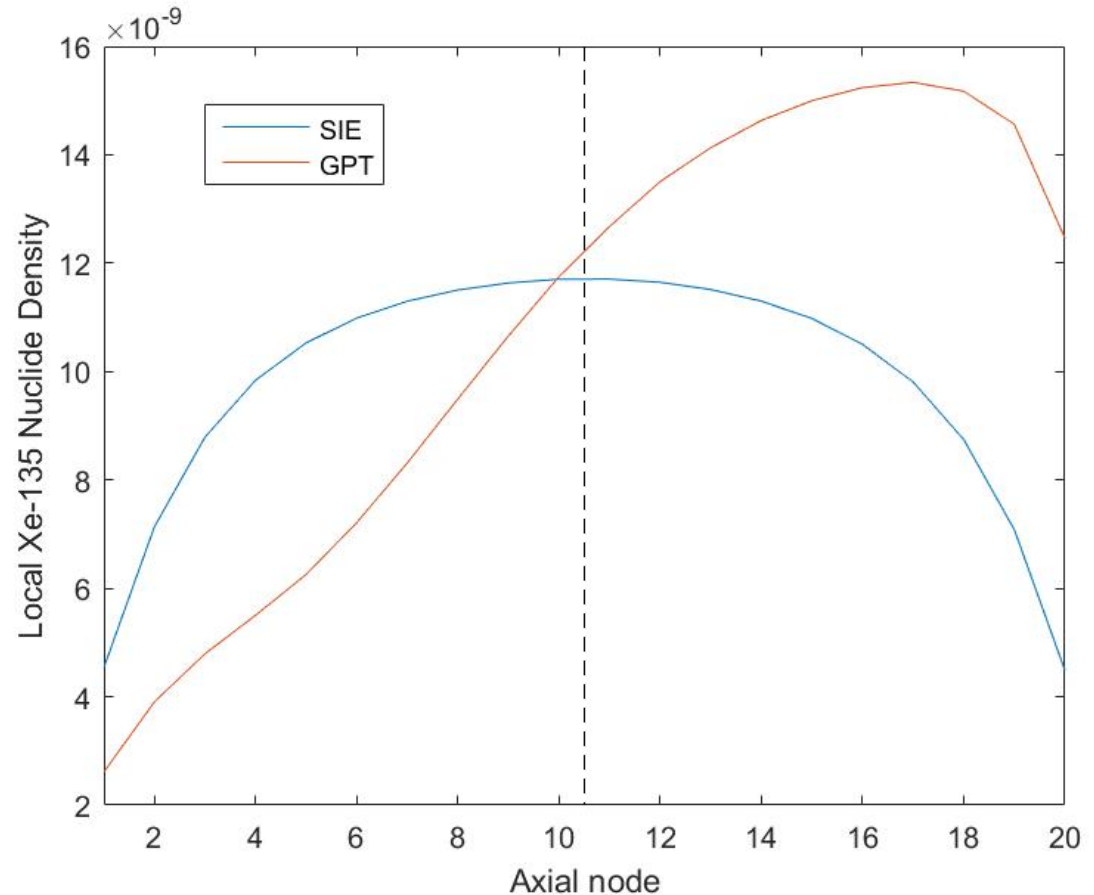
# Results: PWR pin



Pu, U densities remain *mostly* symmetric due to 'sloshing' between Predictor and Corrector

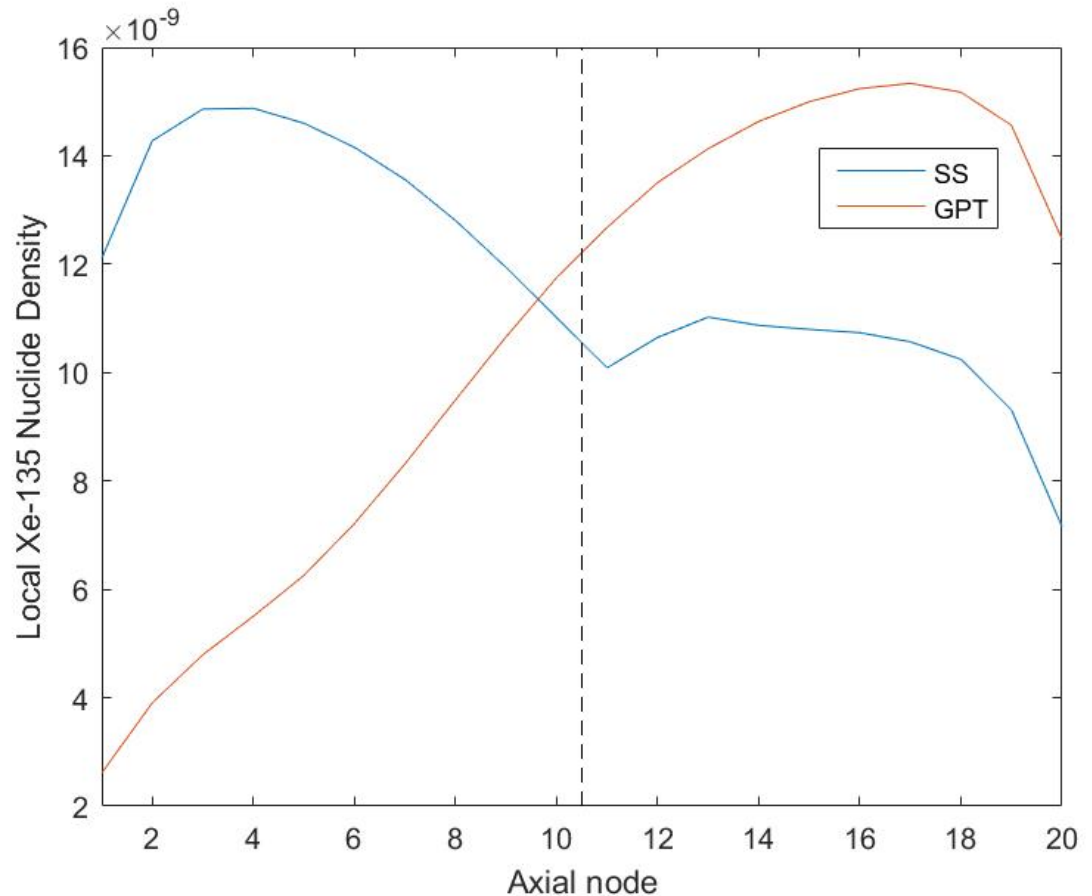
# Results: PWR pin

- Comparison against 5 day timestep SIE at 310 days
- Unfortunately, the perturbation scheme appears only conditionally stable!



# Results: PWR pin

- Comparison against 20 day timestep Substep at 310 days
- Appears to be some improvement over Substep





Thank you!

Questions? Suggestions?