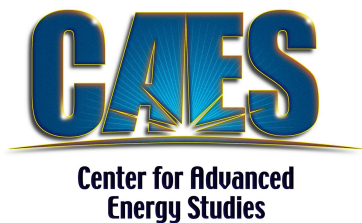




# Serpent Research at Idaho National Laboratory

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**International Serpent  
Users Group Meeting**  
September 26-30, 2016  
Milan, Italy  
INL-STIMS-

## Goal:

Improve simulation of complex nuclear reactors, in particular the Transient Reactor Test Facility (TREAT) and the Advanced Test Reactor (ATR), both at Idaho National Laboratory (INL).

- Coupled Serpent—MOOSE/BISON/MAMMOTH
- Functional Expansion Tallies
- Weighted Delta Tracking



# Coupled Serpent—MOOSE

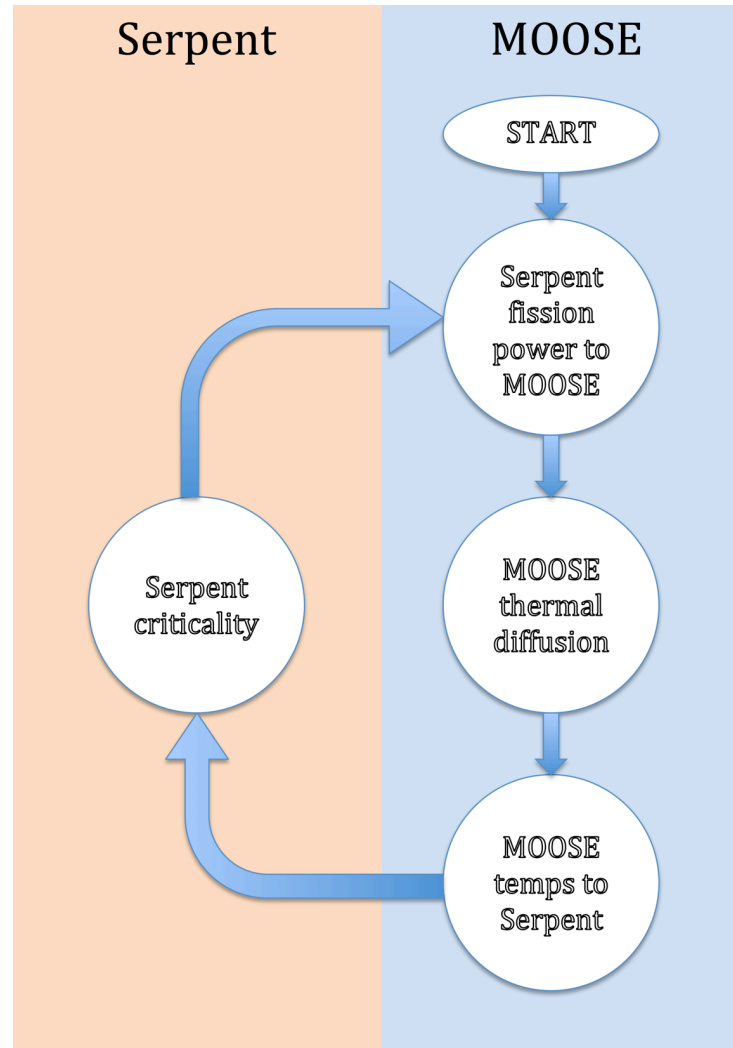
## Built off of prototype provided by Ville Valtavirta

- HeatToMoose: transfers the Serpent fission heat generated, per element volume, to the MOOSE mesh. Simulation must start with initial “guess,” which can be accomplished by running standalone Serpent.
- ElementTransfer: averages the MOOSE temperature solution field for each element, transforms this to an OpenFOAM mesh, and transfers this to the Serpent interface input file.
- RunSerpent: runs the Serpent calculation and updates the fission heat distributions.



# Coupled Serpent—MOOSE II

## Process

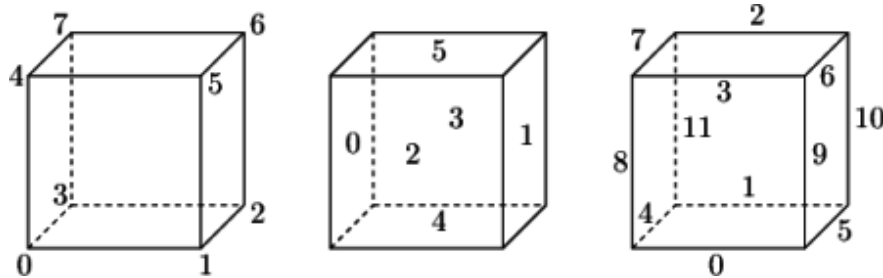




# Coupled Serpent—MOOSE III

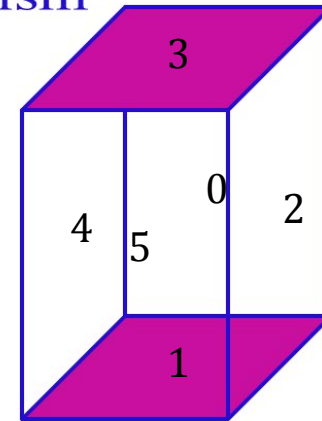
## Meshes

### Serpent



### MOOSE


#### Rectangular Prism



OpenFOAM hexahedron vertices, faces, and edge numbering

Four separate files:

- points
- faces
- owners
- neighbors

*GeneratedMesh* with dim=3 and  Idaho National Laboratory  
elem\_type=PRISM6, with sides numbered

# Coupled Serpent—MOOSE IV

## Code Modification

- Protect header files from multiple definitions;
- Extern global variables declared in header files;
- Create C file to define those global variables;
- Change main to cmain;
- #define OPEN MP and NO GFX MODE;
- Create DATA EXT MODE option;
- Add extern clause to all \*.c and \*.h files;
- Fix Newton's method bug in TMS.

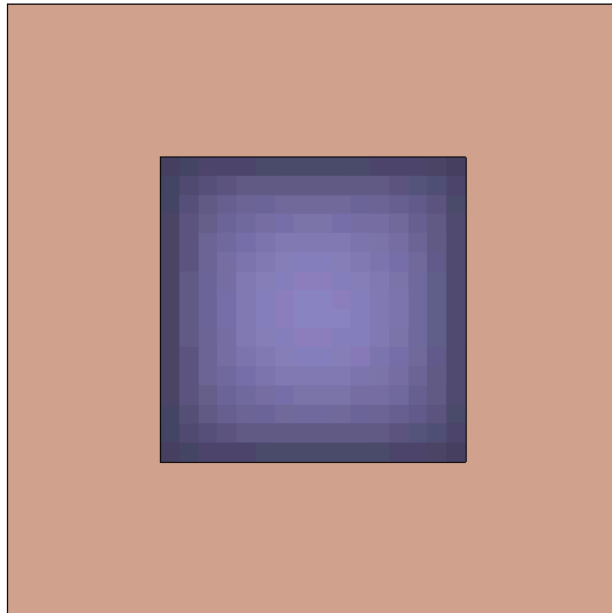
## Compiling Options

- Makefile
- In src/ directory



# Coupled Serpent—MOOSE V

## Single Fuel Pin



Fuel		Water	
Isotope	% mass	Isotope	%mass
$^{235}\text{U}$	2.9971	$^1\text{H}$	66.6667
$^{238}\text{U}$	85.153	$^{16}\text{O}$	33.3333
$^{16}\text{O}$	11.85		
5 $\text{cm}^2$ square		10 $\text{cm}^2$ square	

# Coupled Serpent—MOOSE VI

## Single Fuel Pin Results

	Standalone Serpent 2.1.26	Coupled Serpent—MOOSE
$k_{eff}$ (analog)	0.20592 +/- 0.00080	0.20535 +/- 0.00085
$k_{eff}$ (implicit)	0.20598 +/- 0.00060	0.20542 +/- 0.00061
Transport comp. time	20.9	20.3



# Functional Expansion Tallies

## What are Functional Expansion Tallies (FETs)?

- Typically, Monte Carlo codes score tallies in spatial bins arranged in some type of regular mesh.
- Rather than score tallies in a finite number of bins, FETs score the contribution of each tally to a handful coefficients which correspond to continuous basis functions.



# Functional Expansion Tallies II

## Why implement FETs in Serpent?

- Structured to unstructured mesh conversion
- Only have to store a handful of coefficients
- Data transfer speed up
- Implemented successfully in OpenMC
- Continuous!



# Functional Expansion Tallies III

## Legendre Polynomials

The Legendre polynomials  $P_i$  are defined for integers  $i \geq 0$  as

$$P_i(z) = \sqrt{\frac{2i+1}{2}} \sum_{k=0}^i \binom{i}{k} \binom{-i-1}{k} \left(\frac{1-z}{2}\right)^k \quad (1)$$

where the first two factors in parenthesis are binomial coefficients and the last factor is a real number.



# Functional Expansion Tallies IV

## Zernike Polynomials

The Zernike polynomials  $Z_j = Z_n^m$  are implemented for even  $n - m$  and  $n \geq m$  as

$$Z_n^m(r, \theta) = \begin{cases} \sqrt{2(n+1)} R_n^m(r) \cos(m\theta) & \text{for } m > 0 \\ \sqrt{2(n+1)} R_n^{-m}(r) \sin(-m\theta) & \text{for } m < 0 \\ \sqrt{n+1} R_n^0(r) & \text{for } m = 0 \end{cases} \quad (2)$$

$$R_n^m(r) = \sum_{k=0}^{\frac{n-m}{2}} (-1)^k \binom{n-k}{k} \binom{\frac{n-m}{2}-k}{k} r^{n-2k}$$

where the second and third factors in parenthesis are binomial coefficients.





# Functional Expansion Tallies V

## Calculating the coefficients

From the orthogonality of Zernike and Legendre polynomials, the product  $Z_j(r, \theta)P_i(z)$  satisfies

$$\int_{-1}^1 dz \int_0^1 dr \int_0^{2\pi} d\theta \left( Z_j(r, \theta) P_i(z) \right) \left( Z_{j'}(r, \theta) P_{i'}(z) \right) = \delta_{i,i'} \delta_{j,j'} \quad (3)$$

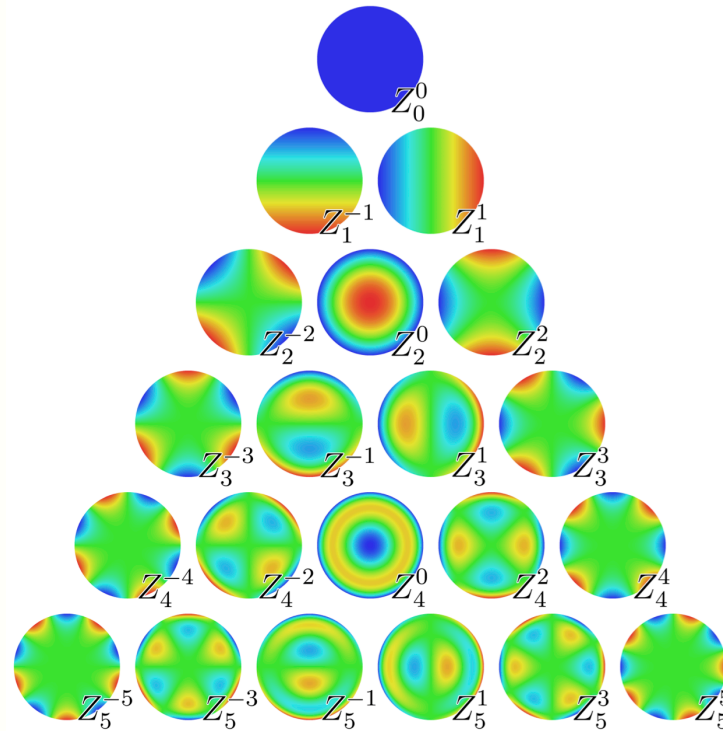
where  $\delta$  is the Kronecker delta function. Note that the forms of the polynomials defined in Eqs. [2] and [1] are normalized so that their inner products are one. The constants  $c_{ij}$  can then be obtained from

$$c_{ij} = \int_{-1}^1 dz \int_0^1 dr \int_0^{2\pi} d\theta f(r, \theta, z) Z_j(r, \theta) P_i(z) \quad (4)$$



# Functional Expansion Tallies VI

## First 15 Zernike Polynomials



"The first 15 Zernike polynomials, ordered vertically by radial degree and horizontally by azimuthal degree" by Zom-B is licensed under CC BY 3.0

# Functional Expansion Tallies VII

## Multiphysics Interface

- New to Serpent 2
- Standardized input and output file formats for reading in temperature/data and writing fission power
- Support for regular mesh, tetrahedral mesh, unstructured mesh, and more
- Type 3 interface 'User Defined Functional Dependence', allows a user to pass in an arbitrary number of parameters and modify userifc.c, the source file that interprets the parameters



# Functional Expansion Tallies VIII

## Reading temperature and density data

---

```
function USERIFC(*f, x, y, z, nz, nr, c0,1, . . . , cij, . . . , cnz-1, nr)
    *f = 0
    i = 0
    for i < nz do
        j = 1
        for j ≤ nr do
            *f = *f + cij × Zj(r(x,y), θ(x,y)) × Pi(z)
        end for
    end for
end function
```

---

Outline of UserIFC() function using  $nz$  Legendre polynomials and  $nr$  Zernike polynomials. Note that  $*f$  is a pointer to either temperature or density at location  $(x,y,z)$ .



# Functional Expansion Tallies IX

## Writing fission power data

---

```
function SCOREINTERFACEPOWER(E, w, nz, nr, x, y, z)
    i = 0
    for i < nz do
        j = 1
        for j ≤ nr do
            E = E ×  $Z_j(r(x, y), \theta(x, y)) \times P_i(z)$ 
            AddScoreToBin(E, w, i, j)
        end for
    end for
end function
```

---

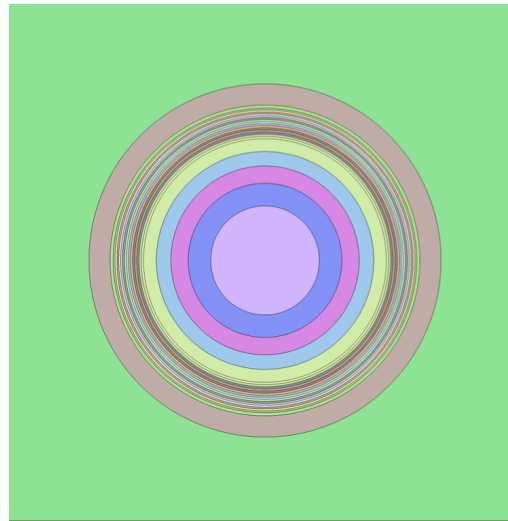
Modified ScoreInterfacePower() function using *nz* Legendre polynomials and *nr* Zernike polynomials. The coefficients are stored in  $nz \times nr$  bins.



# Functional Expansion Tallies X

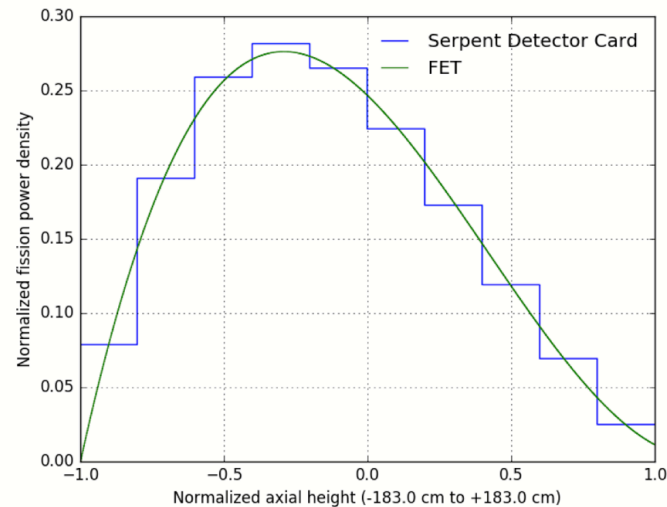
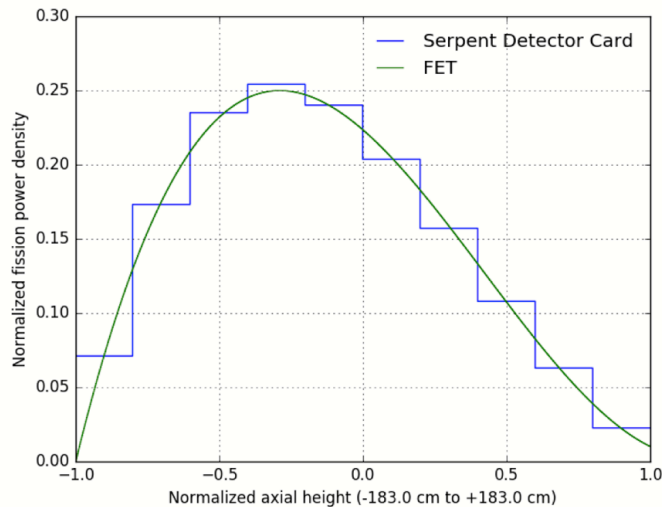
## Serpent Detector Cards

The fuel pin was partitioned into 10 equal-width axial zones and 20 radial zones. The functional expansion tally was carried out to 5<sup>th</sup> order Legendre and 4<sup>th</sup> order Zernike polynomials.



# Functional Expansion Tallies XI

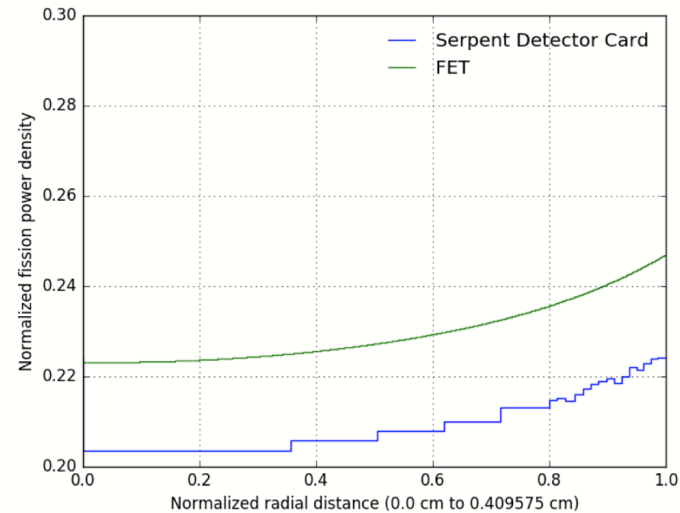
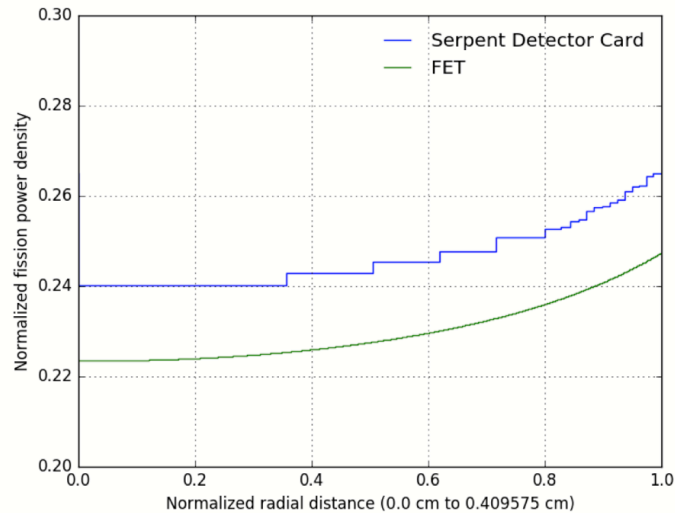
## Axial Fission Power Distribution Comparison



(Left) Fission power density along centerline of fuel pin. (Right) Fission power density along outer edge of fuel pin.

# Functional Expansion Tallies XII

## Radial Fission Power Distribution Comparison



Fission power density at axial midpoint of fuel pin. (Left) Immediately before detector discontinuity. (Right) Immediately after detector discontinuity.





# Weighted Delta Tracking

## Woodcock Delta Tracking

Choose  $g(x)$  using the **majorant cross-section**, the maximum cross-section for our region of interest  $\mathbb{V}$ .

$$\Sigma_{\text{maj}} \equiv \max_{\mathbf{r} \in \mathbb{V}} \{\Sigma_t(\mathbf{r})\}$$

Sample from

$$s(\xi) = F^{-1}(\xi) = -\frac{1}{\Sigma_{\text{maj}}} \ln \xi$$

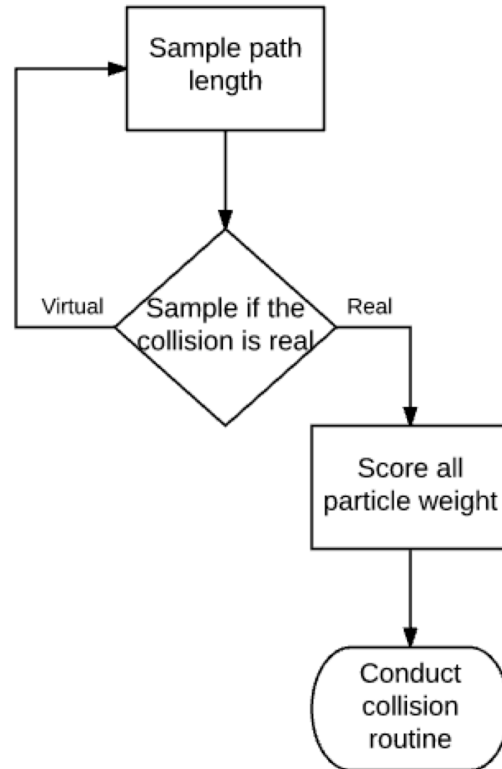
And accept with probability

$$\begin{aligned} P_{\text{real}} &= \frac{f(x)}{Mg(x)} = \frac{\Sigma_t e^{-\Sigma_{\text{maj}} s}}{\Sigma_{\text{maj}} e^{-\Sigma_{\text{maj}} s}} \\ &= \frac{\Sigma_t}{\Sigma_{\text{maj}}} \end{aligned}$$



# Weighted Delta Tracking II

## Woodcock Delta Tracking Process



# Weighted Delta Tracking III

## Woodcock Delta Tracking Advantages/Disadvantages

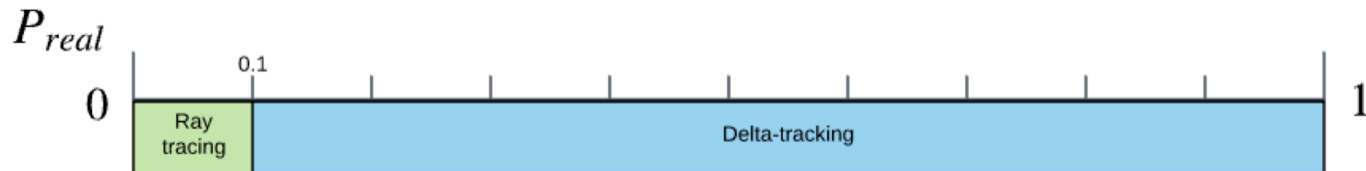
### Benefits:

- ▶ Can sample across multiple geometric regions.
- ▶ No calculation of boundary distance required.

### Downsides:

- ▶ Lots of virtual collisions when  $\Sigma_t \ll \Sigma_{\text{maj}}$
- ▶ Heavy absorbers
- ▶ Can't use track length estimator for flux

Serpent 2 uses a combination of ray tracing and delta-tracking [2]



# Weighted Delta Tracking IV

## Implicit Events

Statistical processes can be replaced by using the expected value of the outcome:[1]

$$E[x] = x_1 p_1 + x_2 p_2 \dots + x_n p_n$$

Implicit capture:

$$\begin{aligned} E[w_f] &= w_{f,\text{scattering}} P_{\text{scattering}} + w_{f,\text{absorption}} P_{\text{absorption}} \\ &= w_i P_{\text{scattering}} \end{aligned}$$

$$\begin{aligned} S_{\text{capture}} &= E[w_i - w_f] \\ &= E[w_i] - E[w_f] \\ &= w_i - w_i P_{\text{scattering}} \\ &= w_i (1 - P_{\text{scattering}}) \end{aligned}$$



# Weighted Delta Tracking V

## Weighted Delta Tracking

Weighted delta-tracking replaces the rejection sampling of delta-tracking with a weight reduction [3].

$$E[w_f] = w_{f,\text{real}}P_{\text{real}} + w_{f,\text{virt}}P_{\text{virt}}$$

For an absorption event:

$$\begin{aligned} E[w_f] &= w_{f,\text{real}}P_{\text{real}} + w_{f,\text{virt}}P_{\text{virt}} \\ &= 0 + w_iP_{\text{virt}} \\ &= w_i(1 - P_{\text{real}}) \\ &= w_i \left( 1 - \frac{\Sigma_t}{\Sigma_{\text{maj}}} \right) \end{aligned}$$



# Weighted Delta Tracking VI

## WDT Scattering

Application of the expectation value to scattering:

$$\begin{aligned} E[w_f] &= w_{f,\text{real}} P_{\text{real}} + w_{f,\text{virt}} P_{\text{virt}} \\ &= w_i P_{\text{real}} + w_i P_{\text{virt}} \\ &= w_i (P_{\text{real}} + 1 - P_{\text{real}}) \\ &= w_i \end{aligned}$$

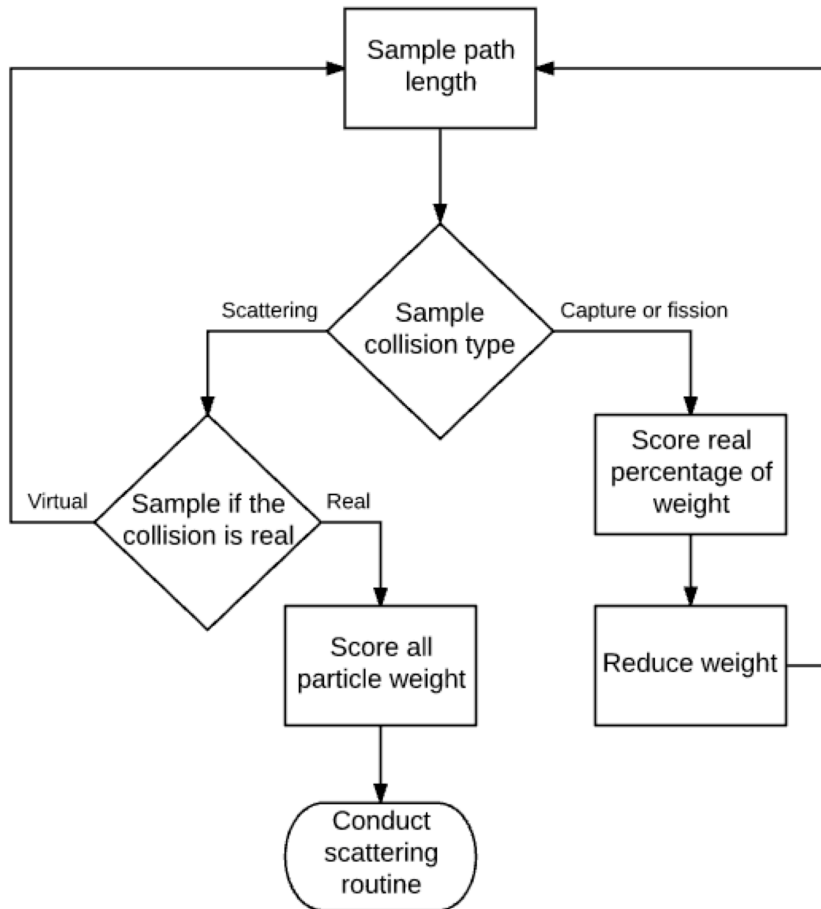
This doesn't make any physical sense. In reality there are two portions of the weight: one that undergoes a real event and one that undergoes a virtual collision. Particle must be split at every scattering event.

Instead, reintroduce rejection sampling into the scattering routine to model virtual collisions explicitly.



# Weighted Delta Tracking VII

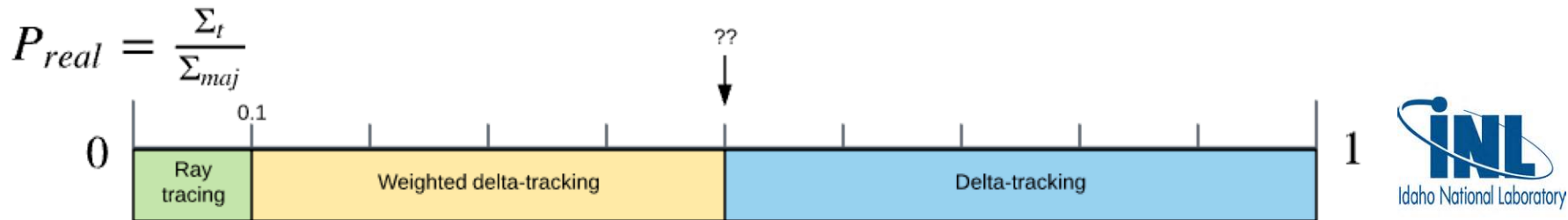
## WDT Process



# Weighted Delta Tracking VIII

## WDT Scattering Summary

- ▶ Requires a rouletting routine.
- ▶ Moves collision scoring to after collision type sampling.
- ▶ Scattering collisions that are determined to be virtual cost more than normal delta-tracking virtual collisions.
- ▶ Weight reduction when  $\Sigma_t \rightarrow \Sigma_{maj}$  generates many low weight particles.





# Weighted Delta Tracking IX

## Homogenous TREAT fuel cell

- ▶ Eleven energy groups.
- ▶ 100 000 source neutrons, 5 inactive and 100 active cycles
- ▶ Interested in cross-section FOM.
- ▶ Run with and without WDT, compare FOM.
- ▶ Looked at  $\Sigma_t$ ,  $\Sigma_c$ ,  $\Sigma_a$ , and  $\Sigma_{\text{scatter}}$  ( $P_0$ ,  $P_1$ , and  $P_2$ ).

Figure of merit:

$$\text{FOM} = \frac{1}{\sigma(\hat{x})^2 T}$$

FOM ratio:

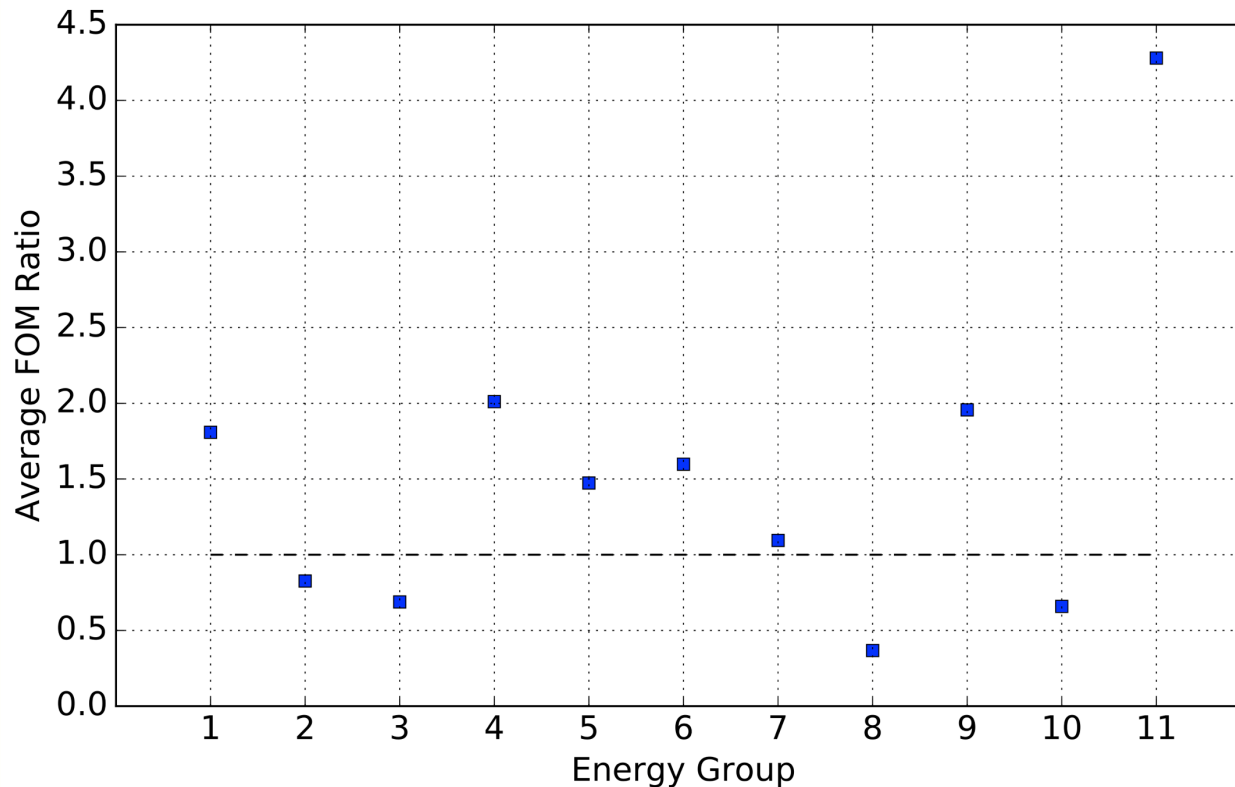
$$\text{Ratio} = \frac{\text{FOM}_{\text{WDT}}}{\text{FOM}_{\text{noWDT}}}$$



# Weighted Delta Tracking X

## Homogenous TREAT fuel cell FOM

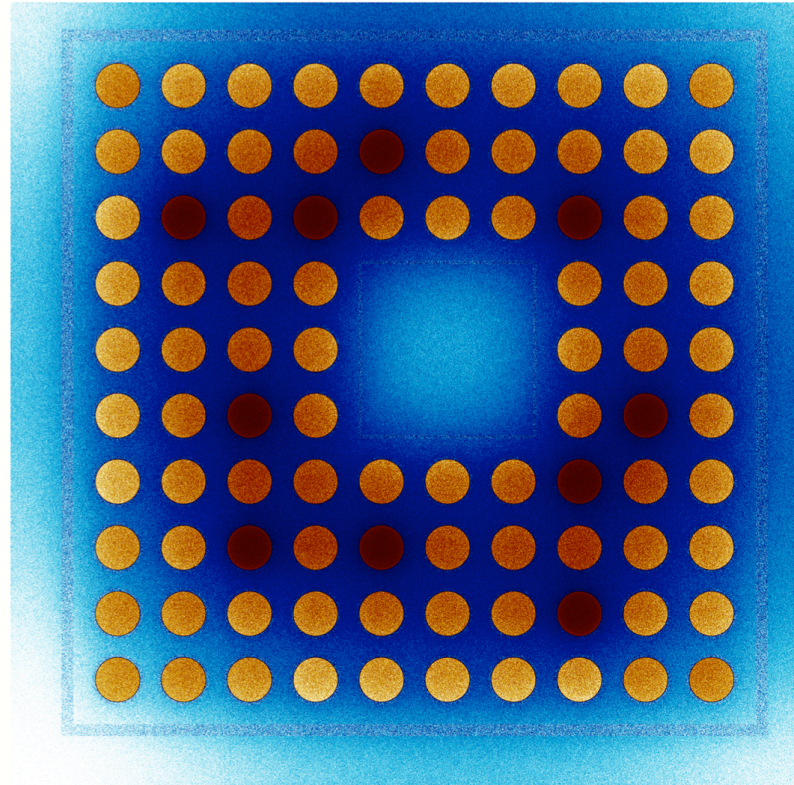
Average FOM Ratio for cross-sections by energy group



# Weighted Delta Tracking XI

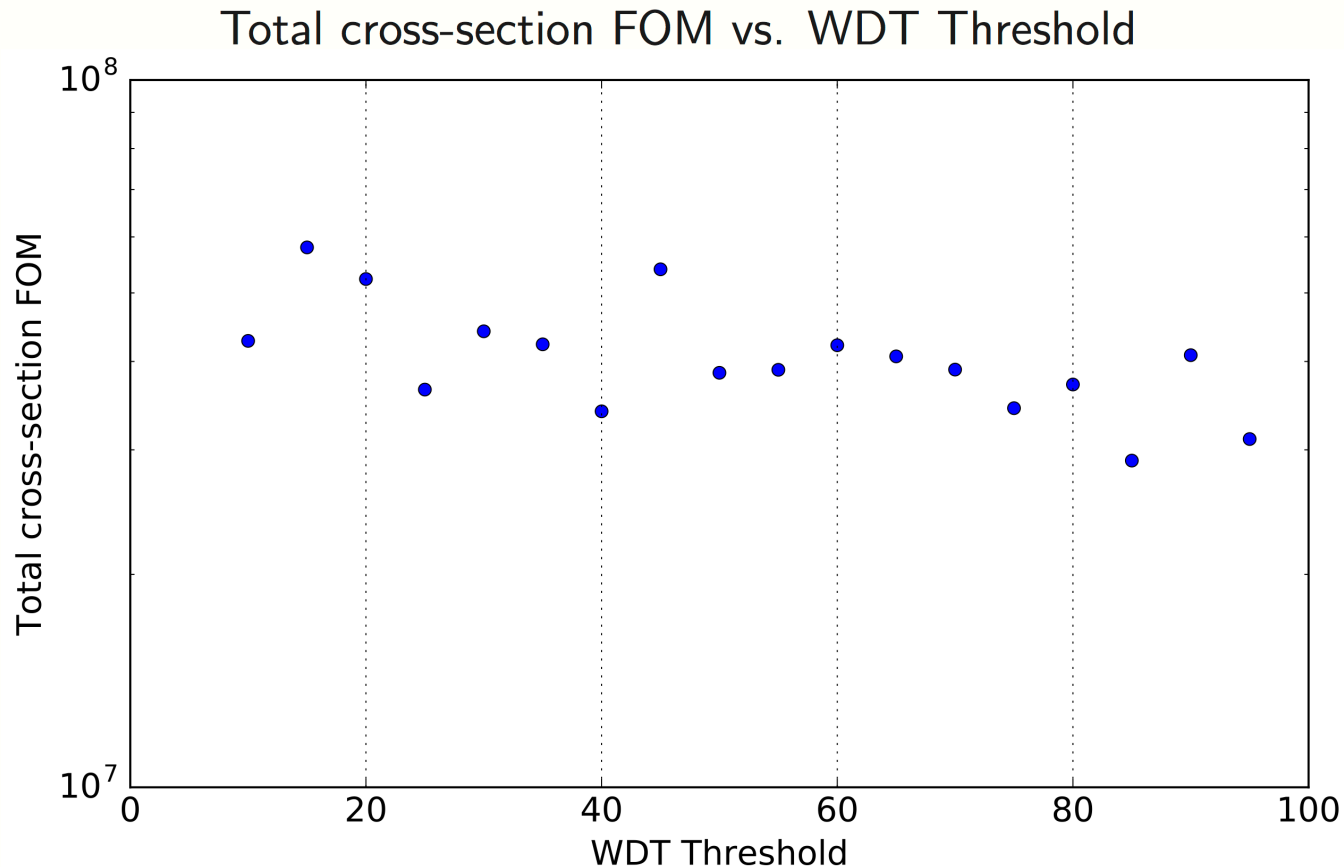
## BWR

- ▶ Gd pins
- ▶ 1.295 cm pitch
- ▶ 25% void fraction
- ▶ 20 000 neutrons, 10 inactive, 500 active cycles



# Weighted Delta Tracking XII

## BWR FOM



# Future Work

## Serpent—BISON coupling

- Analyze Serpent—MOOSE coupled fission, thermal
- FETs implemented, optimized in coupled Serpent
- Serpent FETs → BISON
- BISON functional expansions → Serpent
- Serpent—BISON coupling optimization
- V&V

## Weighted Delta Tracking (WDT)

- Replace some surface tracking with WDT
- Test on TREAT



# Acknowledgements

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