

Serpent meeting 2016  
Polimi, Milano, Italy,  
Sept. 26 – 30, 2016

# **Nuclear data uncertainty quantification in SFR, LFR, and MSR via Monte Carlo perturbation theory.**

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# Nuclear Data S/U via Monte Carlo Perturbation

- Nuclear data represent the starting point of “any” neutronics calculation (*set acelib “...xsdata” ...*)
- Experimental uncertainties and model approximations affect our knowledge of nuclear data
- Interest in sensitivity and uncertainty analysis within the nuclear community is continuously increasing
- The implementation of Monte Carlo Perturbation Theory has been the focus of recent research activities
- Main goal: avoid any discretization for the solution of the adjoint problem (no mesh, no energy grid, no angular discretization)



- Monte Carlo Sensitivity/Perturbation in Serpent  
(short version!)
- Verification of Serpent sensitivity calculations against direct perturbation
- Analysis of the coolant void reactivity coefficient in LFR and SFR
- Results of the uncertainty quantification studies in ALFRED, ESFR, MSFR, MOSART, REBUS-3700

# Generalized perturbation theory capabilities in Serpent

Generalized Perturbation Theory (GPT) allows calculating the effects of several different perturbations within a single run, usually in the form of sensitivity coefficients:

$$S_x^R \equiv \frac{dR/R}{dx/x}$$

Sensitivity/perturbation capabilities were implemented in SERPENT for generalized response functions [\*].

[\*] M Aufiero, A Bidaud, M Hursin, J Leppänen, G Palmiotti et al. "A collision history-based approach to sensitivity/perturbation calculations in the continuous energy Monte Carlo code serpent." *Annals of Nuclear Energy*, 85 (2015).



# Generalized Perturbation Theory capabilities

Effect of a perturbation of the parameter  $x$  on the response  $R$  :

$$S_x^R \equiv \frac{dR/R}{dx/x}$$

Considered response functions:

$R = k_{\text{eff}}$  Effective multiplication factor

$R = \frac{\langle \Sigma_1, \phi \rangle}{\langle \Sigma_2, \phi \rangle}$  Reaction rate ratios

$R = \frac{\langle \phi^\dagger, \Sigma_1 \phi \rangle}{\langle \phi^\dagger, \Sigma_2 \phi \rangle}$  Bilinear ratios (Adjoint-weighted quantities)

$R = \frac{E[e_1]}{E[e_2]}$  Something else (see Friday's presentation on Perturbation Theory for acceleration of multiphysics calculations)



# Calculation of $k_{\text{eff}}$ sensitivity to coolant density

We can obtain space-dependent  $k_{\text{eff}}$  sensitivities to coolant density adopting the collision-history approach...

$$S_x^{k_{\text{eff}}} \equiv \frac{dk_{\text{eff}}/k_{\text{eff}}}{dx/x} = E \left[ {}^{(-\gamma)}ACC_x - {}^{(-\gamma)}REJ_x \right] \quad (1)$$

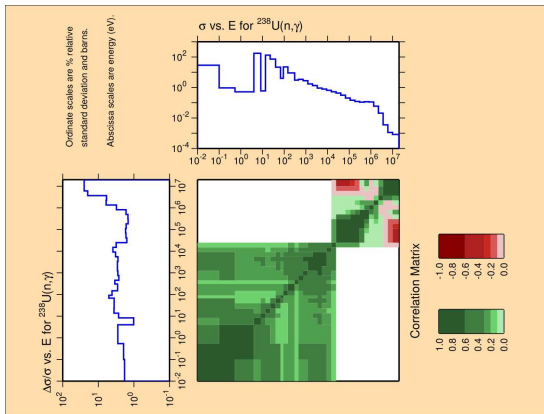
- $x$  represents the coolant density in different zones
- ${}^{(-\gamma)}ACC_x$  and  ${}^{(-\gamma)}REJ_x$  represent the accepted and rejected neutron collisions history of the particles
- $\gamma$  is the number of latent generations adopted in the Iterated Fission Probability estimators for the convergence of the adjoint weighting.



# Uncertainty propagation

First-order uncertainty propagation formula (sandwich rule)

$$\text{Var}[R] = \underline{S_x^R} \text{Cov}[\underline{x}] \left( \underline{S_x^R} \right)^T$$



# Uncertainty propagation

First-order uncertainty propagation formula (sandwich rule)

$$\text{Var} [R] = \underline{S_x^R} \text{Cov} [\underline{x}] \left( \underline{S_x^R} \right)^T$$

COMMARA-2.0 Neutron Cross Section Covariance Library

(thanks to Pino Palmiotti @INL for precious suggestions!)



# The coolant density reactivity coefficient in SFR & LFR

- The coolant density effect represents one of the main reactivity feedback in LFR and SFR
- Its accurate (space-dependent) calculation is important for a correct evaluation of the dynamics of these systems
- Monte Carlo neutron transport simulations are commonly adopted for the study of LFR and SFR [a,b]

[a] L. Buiron et al. "Evaluation of large 3600mwth sodium-cooled fast reactor neutronic oecd benchmarks." In: PHYSOR 2014, Kyoto, Japan, Sep. 28 - Oct. 3, 2014 (2014).

[b] G. Grasso et al. "The core design of ALFRED, a demonstrator for the european lead-cooled reactors." Nuclear Engineering and Design, 278: pp. 287–301 (2014).



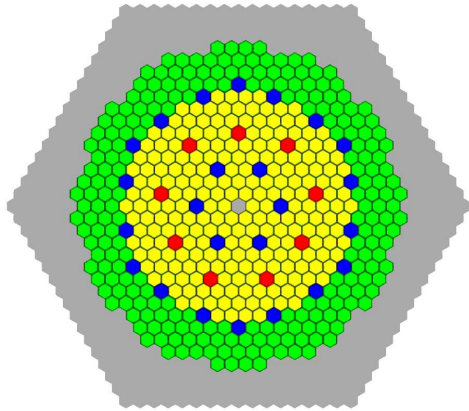
# The coolant density reactivity coefficient in SFR & LFR

- Using MC, reactivity feedbacks are usually calculated via direct perturbations, i.e., comparing the effective multiplication factor of two separate Monte Carlo runs.
- This approach is inefficient and requires a separate calculation per each spatial zone
- Here we use sensitivity/perturbation calculations in Monte Carlo to analyze the coolant reactivity coefficient in SFR & LFR



# Large sodium-cooled fast reactor

OECD/NEA SFR Benchmark Task Force

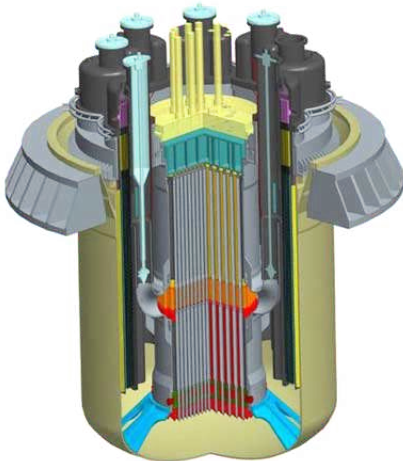


225 inner and 228 outer MOX FAs. Axially variable Pu content  
2 radial  $\times$  5 axial zones considered for Na reactivity calculations

# Advanced Lead Fast Reactor European Demonstrator

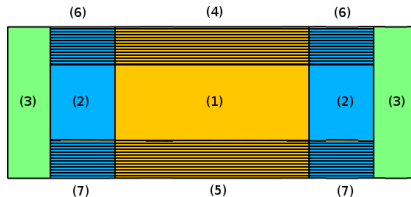
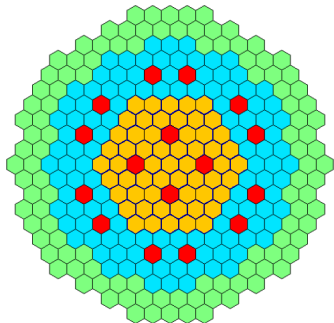
Developed within the European FP7 LEADER project

ALFRED, is a small-size (300MWth) pool-type LFR.



# Advanced Lead Fast Reactor European Demonstrator

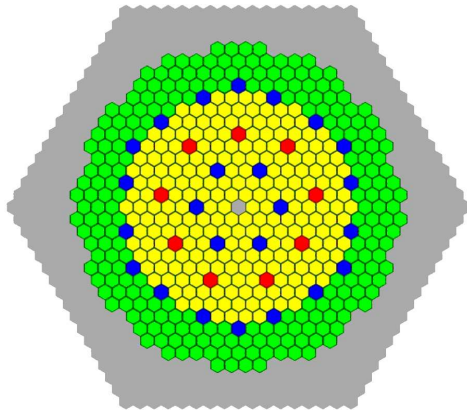
Developed within the European FP7 LEADER project



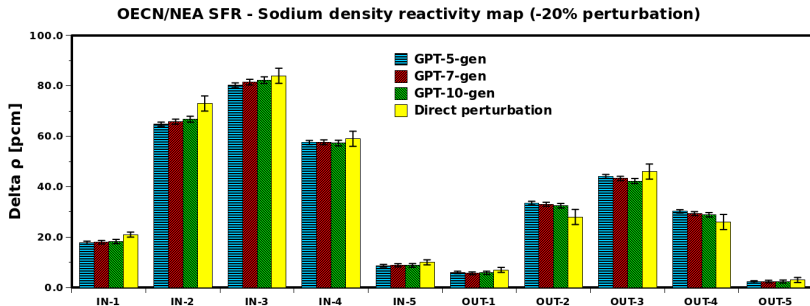
171 FAs are subdivided into two radial zones with different plutonium fractions

7 different zones were considered for the Pb density reactivity coefficient calculations

# Verification of the perturbation calculations: SFR



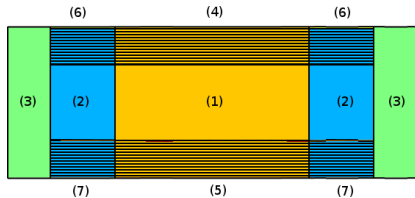
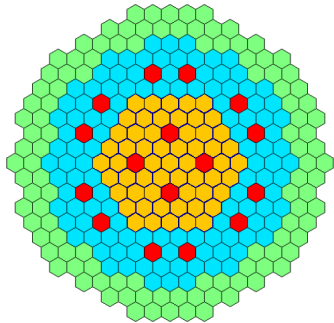
# Verification of the perturbation calculations: SFR



**Positive void effect in all the zones:**

Decreasing the sodium density in each of the  $2 \times 5$  zones leads to a reactivity increase

# Verification of the perturbation calculations: ALFRED



# Verification of the perturbation calculations: ALFRED

**Table:** Lead density  $k_{\text{eff}}$  sensitivity coefficient in the considered ALFRED reactor core zones. Comparison between sensitivity/perturbation and direct perturbation Serpent results.

Fuel region	Axial zone	$k_{\text{eff}}$ lead density sensitivity coefficient*		
		Direct Pert. #	Pert. Theory	Difference
Inner	Active	$-7.54 \pm 0.13$	$-7.75 \pm 0.30$	-0.21
Outer	Active	$2.69 \pm 0.13$	$2.68 \pm 0.31$	-0.01
Reflector	-	$4.34 \pm 0.13$	$4.34 \pm 0.10$	0.00
Inner	Lower plenum	$4.86 \pm 0.13$	$4.85 \pm 0.10$	-0.01
Outer	Lower plenum	$7.20 \pm 0.13$	$7.70 \pm 0.08$	<b>0.50</b>
Inner	Upper plenum	$5.64 \pm 0.13$	$5.55 \pm 0.13$	-0.09
Outer	Upper plenum	$6.96 \pm 0.13$	$6.79 \pm 0.15$	-0.17
Total	-	$30.4 \pm 0.13$	$29.6 \pm 0.53$	-0.8

\*  $\text{pcm}/(\% \text{ lead density}) \pm 1 \text{ sigma}$

# Direct pert. with 10% lead density variation

All negative void reactivity except inner active region

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# Decomposition of the Pb density sensitivity coefficient

The lead density sensitivity coefficient is the result of the sum of different opposite effects.

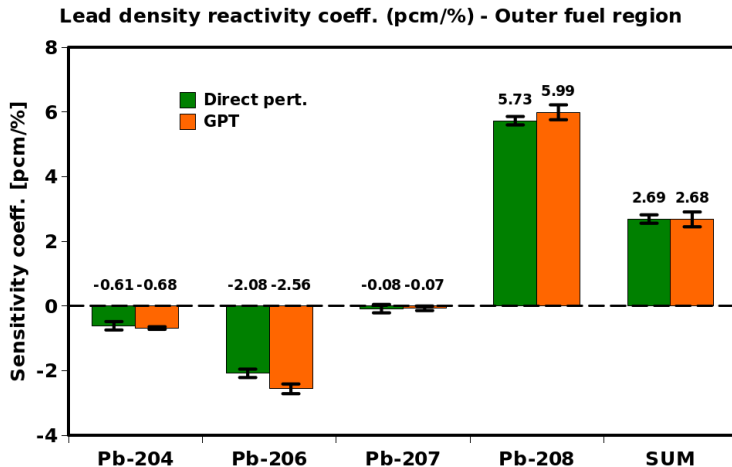
For a better understanding of the phenomena involved, the outer fuel Pb density coefficient was analyzed via decomposition of the reactivity effect by:

- Isotope ( $^{204}\text{Pb}$ ,  $^{206}\text{Pb}$ ,  $^{207}\text{Pb}$ ,  $^{208}\text{Pb}$ )
- Reaction (elastic, inelastic, capture)
- Energy group (33 groups ECCO structure)



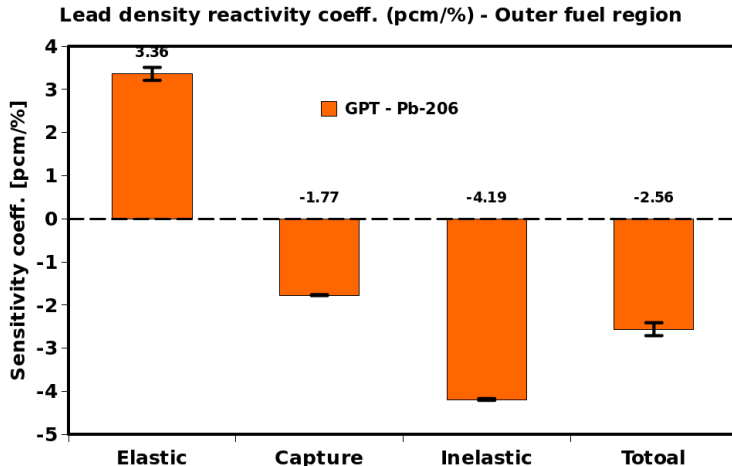
# Decomposition of the Pb density sensitivity coefficient

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# Decomposition of the Pb density sensitivity coeff. – Pb-206

The lead density sensitivity coefficient is the result of the sum of different opposite effects.



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# Decomposition of the Pb density sensitivity coefficient

The lead density sensitivity coefficient is the result of the sum of different opposite effects.

**Elastic:** main impact on leakage term, small spectral effect  
slow IFP convergence due to modification of the fission source

**Inelastic:** negative spectral effect (reduces the average number of neutrons produced per absorption in the fuel)

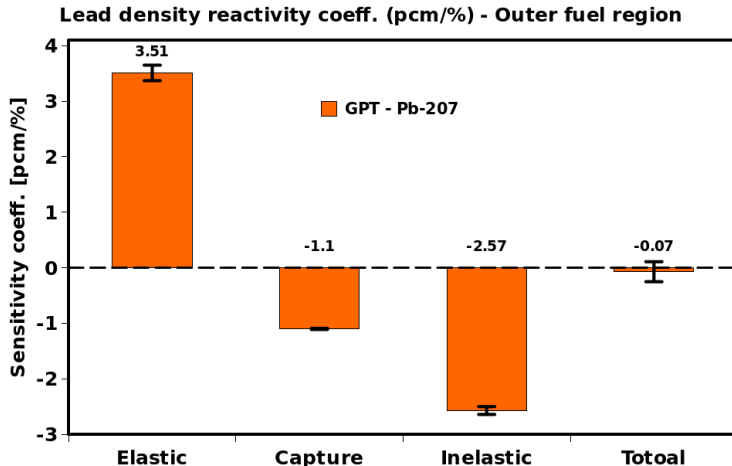
**Capture:** direct negative impact on neutron balance

**(n, xn):** negligible effect



# Decomposition of the Pb density sensitivity coeff. – Pb-207

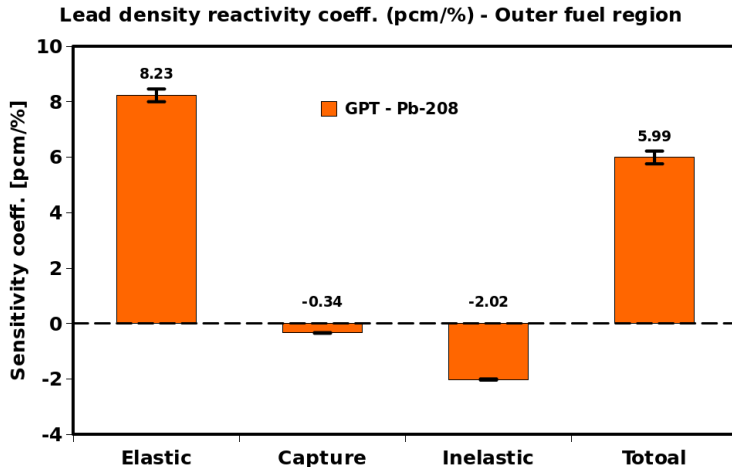
The lead density sensitivity coefficient is the result of the sum of different opposite effects.



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# Decomposition of the Pb density sensitivity coeff. – Pb-208

The lead density sensitivity coefficient is the result of the sum of different opposite effects.



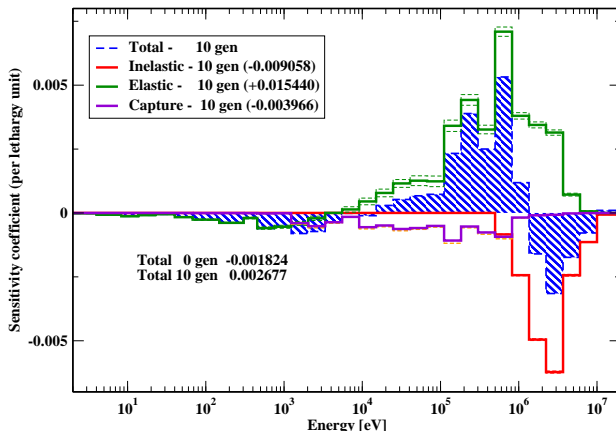
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# Decomposition of the Pb density sensitivity coefficient

The lead density sensitivity coefficient is the result of the sum of different opposite effects.

## ALFRED - Lead density sensitivity coefficient

Outer fuel active region



# Notes on coolant reactivity map via MC Pert. Thoery

- Serpent sensitivity calculations show good agreement against direct perturbation for both the SFR and LFR cases
- A single extended Serpent run to get the sensitivity coefficients for all the regions
- Negative void reactivity in ALFRED  
positive void reactivity in OECD/NEA SFR
- Coolant density reactivity coefficient is the result of the sum of different opposite effects.
- Energy-Isotope-Reaction decomposition can be obtained at no additional cost



# Notes on coolant reactivity map via MC Pert. Thoery

- Serpent sensitivity calculations show good agreement against direct perturbation for both the SFR and LFR cases
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positive void reactivity in OECD/NEA SFR
- Coolant density reactivity coefficient is the result of the sum of different opposite effects.
- Energy-Isotope-Reaction decomposition can be obtained at no additional cost

Uncertainty quantification is just a couple of matrix-vector products ahead...

$$\text{Var} [R] = \underline{S_x^R} \text{Cov} [\underline{x}] \left( \underline{S_x^R} \right)^T$$



# Uncertainty quantification – ALFRED $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
815 pcm**

**Major contributors (in pcm):**

INELASTIC U238 INELASTIC U238  
522.96  
CAPTURE PU239 CAPTURE PU239  
291.94  
NU PU240 NU PU240  
233.29  
CAPTURE U238 CAPTURE U238  
207.07  
FISSION PU239 FISSION PU239  
187.42  
CAPTURE PU240 CAPTURE PU240  
139.58  
ELASTIC U238 INELASTIC U238  
133.19  
FISSION PU240 FISSION PU240  
131.33  
KHI PU239 KHI PU239  
130.89

INELASTIC FE56 CAPTURE FE56  
-3.5118  
ELASTIC PU239 FISSION PU239  
-5.6756  
ELASTIC U238 CAPTURE U238  
-12.622  
FISSION PU239 CAPTURE PU239  
-29.351



# Uncertainty quantification – ALFRED Pb density coeff. Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total outer Pb density reactivity coeff. uncertainty:  
25 % (1 sigma!!)**

**Major contributors (in %):**

INELASTIC U238 INELASTIC U238  
18.127  
INELASTIC PB206 INELASTIC PB206  
8.9843  
INELASTIC PB207 INELASTIC PB207  
7.3012  
CAPTURE PU239 CAPTURE PU239  
4.8552  
CAPTURE U238 CAPTURE U238  
3.8518  
ELASTIC PB208 ELASTIC PB208  
3.6351  
CAPTURE PB207 CAPTURE PB207  
3.5590  
INELASTIC PB208 INELASTIC PB208  
3.2379  
CAPTURE PB208 CAPTURE PB208  
3.1823

FISSION PU239 CAPTURE PU239  
-0.37053  
ELASTIC U238 CAPTURE U238  
-0.37102  
ELASTIC PU240 INELASTIC PU240  
-0.69637  
ELASTIC PU239 CAPTURE PU239  
-0.82188



# Uncertainty quantification – OECD-NEA SFR $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
1380 pcm**

**Major contributors (in pcm):**

INELASTIC U238 INELASTIC U238  
1205.0  
CAPTURE U238 CAPTURE U238  
279.55  
NU PU240 NU PU240  
228.43  
CAPTURE PU239 CAPTURE PU239  
219.43  
KHI PU239 KHI PU239  
208.23  
FISSION PU239 FISSION PU239  
193.16  
ELASTIC U238 INELASTIC U238  
175.63  
NU U238 NU U238  
154.15  
INELASTIC FE56 INELASTIC FE56  
138.99

ELASTIC NA23 CAPTURE NA23  
-5.8231  
INELASTIC FE56 CAPTURE FE56  
-6.7224  
ELASTIC U238 CAPTURE U238  
-16.078  
FISSION PU239 CAPTURE PU239  
-25.984



# Uncertainty in fast-spectrum Molten Salt Reactors

- REBUS-3700<sup>[a]</sup>

- U-Pu cycle breeder:  $\text{NaCl} - \text{UCl}_3 - \text{PuCl}_3$

[a] Mourougov, A., and P. M. Bokov. "Potentialities of the fast spectrum molten salt reactor concept: REBUS-3700." *Energy conversion and management* 47.17 (2006).

- Molten Salt Fast Reactor (MSFR)<sup>[b]</sup>

- U-233 started:  $\text{LiF} - \text{ThF}_4 - \text{UF}_4$
- TRU started:  $\text{LiF} - \text{ThF}_4 - [\text{TRU}]\text{F}_3$

[b] Merle-Lucotte, E., et al. "Launching the thorium fuel cycle with the Molten Salt Fast Reactor." (2011).

- MOSART<sup>[c]</sup>

- TRU Burner:  $\text{NaF} - \text{LiF} - \text{BeF}_2 - [\text{TRU}]\text{F}_3$

[c] F. Gabrielli, A. Rineiski, V. Ignatiev, O. Feinberg "Benchmark on MOLten Salt Advanced Reactor Transmuter (MOSART): Impact of Minor Actinide Nuclear Data Uncertainty on Integral Reactor Parameters" NEA/EGEMAM-II.



# Uncertainty quantification – REBUS-3700 $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
935 pcm**

**Major contributors (in pcm):**

INELASTIC U238 INELASTIC U238  
731.22

CAPTURE U238 CAPTURE U238  
230.27

ELASTIC U238 INELASTIC U238  
228.94

NU PU240 NU PU240  
206.34

FISSION PU239 FISSION PU239  
186.28

CAPTURE PU239 CAPTURE PU239  
176.39

NU U238 NU U238  
152.42

FISSION PU240 FISSION PU240  
115.50

INELASTIC NA23 INELASTIC NA23  
107.94

ELASTIC U238 CAPTURE U238  
-5.6530

FISSION PU239 CAPTURE PU239  
-6.2497

ELASTIC PU239 FISSION PU239  
-7.0126

ELASTIC NA23 CAPTURE NA23  
-8.4695



# Uncertainty quantification – $^{233}\text{U}$ -Started MSFR $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
1640 pcm**

**Major contributors (in pcm):**

CAPTURE TH232 CAPTURE TH232

1220.5

FISSION U233 FISSION U233

899.06

CAPTURE U233 CAPTURE U233

455.88

NU U233 NU U233

376.06

ELASTIC LI7 ELASTIC LI7

125.17

ELASTIC F19 ELASTIC F19

92.890

ELASTIC TH232 ELASTIC TH232

79.712

CAPTURE F19 CAPTURE F19

72.669

INELASTIC F19 INELASTIC F19

69.498

ELASTIC U233 N $\times$ N U233

-0.27131

ELASTIC LI7 INELASTIC LI7

-1.7293

FISSION TH232 CAPTURE TH232

-21.180

ELASTIC TH232 CAPTURE TH232

-44.133



# Uncertainty quantification – TRU-Started MSFR $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
1040 pcm**

**Major contributors (in pcm):**

CAPTURE TH232 CAPTURE TH232  
722.54

CAPTURE PU239 CAPTURE PU239  
505.03

INELASTIC F19 INELASTIC F19  
281.63

CAPTURE PU240 CAPTURE PU240  
220.74

FISSION PU239 FISSION PU239  
184.59

CAPTURE NP237 CAPTURE NP237  
145.85

NU PU240 NU PU240  
140.43

FISSION PU241 FISSION PU241  
137.15

CAPTURE PU242 CAPTURE PU242  
132.77

FISSION TH232 CAPTURE TH232  
-16.362

ELASTIC TH232 CAPTURE TH232  
-30.245

FISSION PU239 CAPTURE PU239  
-69.163

ELASTIC F19 INELASTIC F19  
-149.35



# Uncertainty quantification – TRU-Burner MOSART $k_{\text{eff}}$

Serpent GPT + ENDF/B-VII & COMMARA-2.0

**Total  $k_{\text{eff}}$  uncertainty from nuclear data:  
945 pcm**

**Major contributors (in pcm):**

FISSION PU239 FISSON PU239

518.57

CAPTURE PU240 CAPTURE PU240

445.94

FISSION PU241 FISSON PU241

439.50

CAPTURE PU242 CAPTURE PU242

265.71

CAPTURE PU239 CAPTURE PU239

229.02

CAPTURE PU238 CAPTURE PU238

158.77

ELASTIC NA23 ELASTIC NA23

144.47

ELASTIC F19 ELASTIC F19

111.84

ELASTIC PU240 ELASTIC PU240

110.99

FISSION AM241 CAPTURE AM241

-1.2260

FISSION PU240 CAPTURE PU240

-11.078

ELASTIC AM241 CAPTURE AM241

-11.710

FISSION PU239 CAPTURE PU239

-176.83



# Summary

- Monte Carlo Perturbation Theory has been implemented in Serpent for generalized response functions (GPT)
- This opens the possibility to perform reactor physics calculations in a more efficient way (e.g., see next two presentations by Eugene and Giorgio)
- Uncertainty quantification has been the territory of deterministic codes for decades... not anymore(?)
- The flexibility of Monte Carlo widens the legacy capabilities of Generalized Perturbation Theory towards continuous (in Energy and Space) sensitivity functions



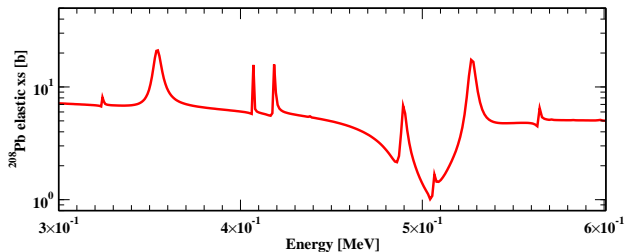
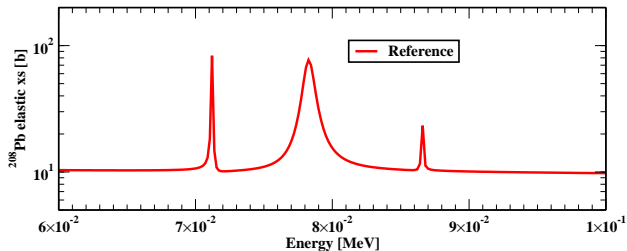
# New applications for Monte Carlo Perturbation Theory

Hints for Friday's presentation

- Monte Carlo eXtended Generalized Perturbation Theory (XGPT): building Reduced Order Models for continuous-energy XS uncertainty propagation
- Jacobian matrix approximation via eigenfunction perturbation theory: accelerating and stabilizing coupled Monte Carlo / CFD multiphysics calculations

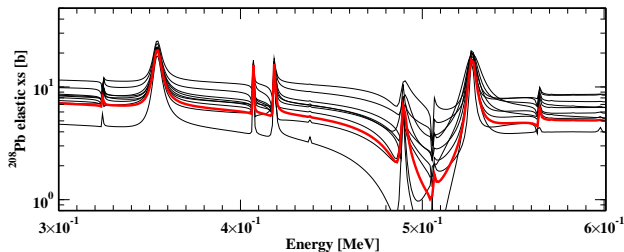
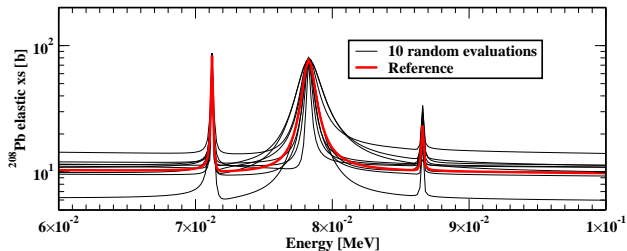


# Realistic representation of uncertainties: random XS



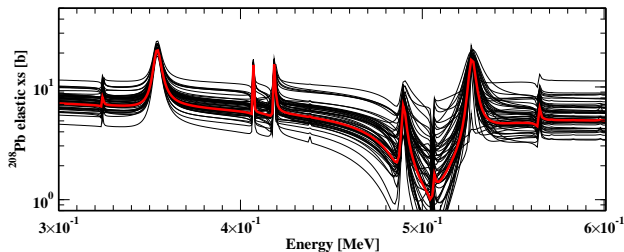
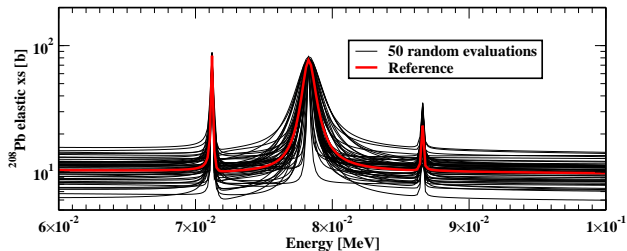
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# Realistic representation of uncertainties: random XS

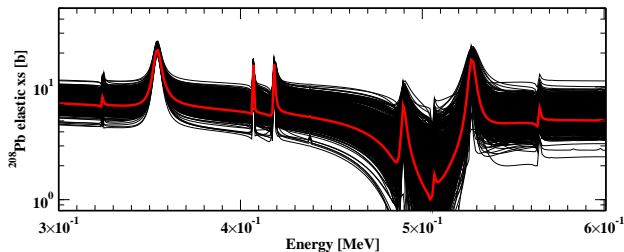
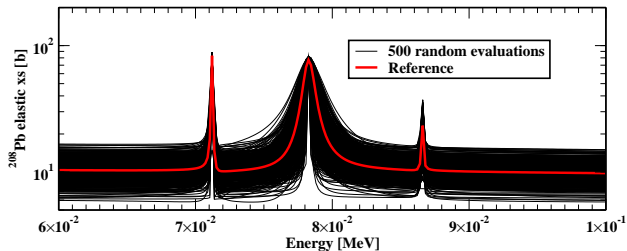


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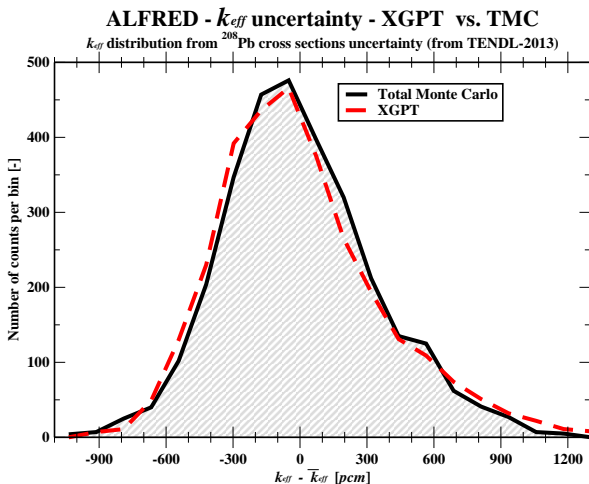
# Realistic representation of uncertainties: random XS



# Realistic representation of uncertainties: random XS



# ALFRED: $^{208}\text{Pb}$ uncertainty analysis via TMC & XGPT



Thank you for the  
attention!

Questions? Suggestions?

