

*Serpent User's group meeting, Berkeley, CA,
November 6-8, 2013*

Monitoring Numerical Stability of Coupled MC codes

D. Kotlyar, E. Shwageraus

**Department of Nuclear Engineering,
Ben Gurion University of the Negev**





- ❑ **Background**
 - ❑ Demonstration how instabilities can occur (PWR/BWR)
- ❑ **Description of coupling schemes**
 - ❑ Definition of a couple scheme
 - ❑ Explicit vs. Implicit
- ❑ **BWR test case**
 - ❑ BOT and SIMP results
 - ❑ Computational requirements of the implicit method
 - ❑ Quantitative stability assessment
- ❑ **Performance of hybrid coupling scheme**
 - ❑ Stability monitoring
 - ❑ Computational requirements
- ❑ **Summary and conclusions**



- ❑ The objective of the coupled MC analysis is to obtain:
 - Nuclide density field as a function of t
 - TH properties as a function of t
- ❑ This non-linear problem is solved by operator splitting
 - Described by 3 coupled equations:
 - Burnup: describes the changes in ND
 - Heat balance equation: computes temperature distribution
 - Eigenvalue neutron transport equation: provides the neutron flux
- ❑ How to couple the independent solutions ?



❑ The explicit **BOT** method

- Neutronic-TH convergence at BOS
- Depletion with BOS (explicit) flux values
- May be unstable due to the numerical explicit coupling scheme

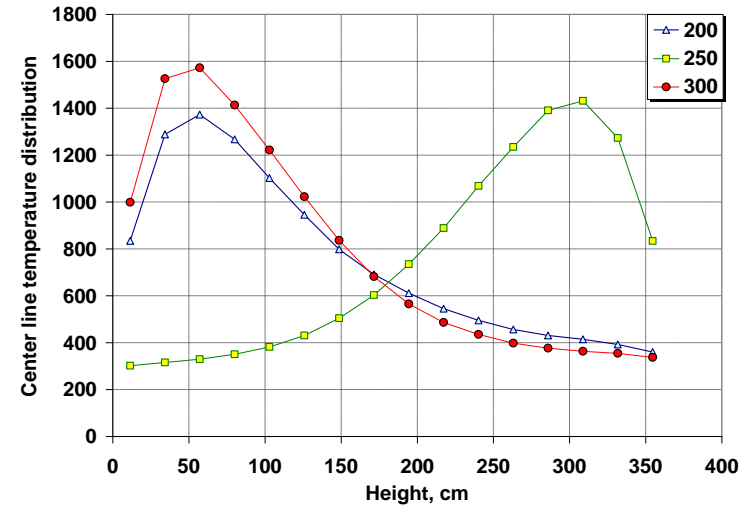
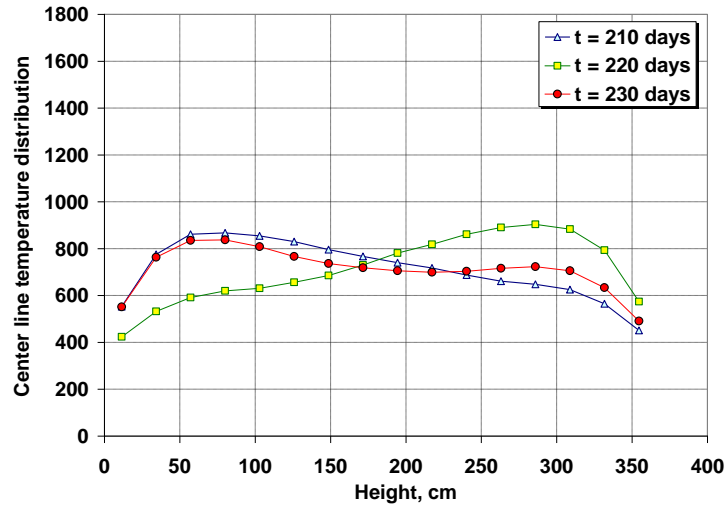
❑ Stochastic Implicit Mid-Point (**SIMP**) method

- Simultaneous convergence of ND and TH fields
- Uses EOS fluxes and thus implicit
- Flux/power/temperatures are time-step averaged quantities
- Proven to be numerically stable

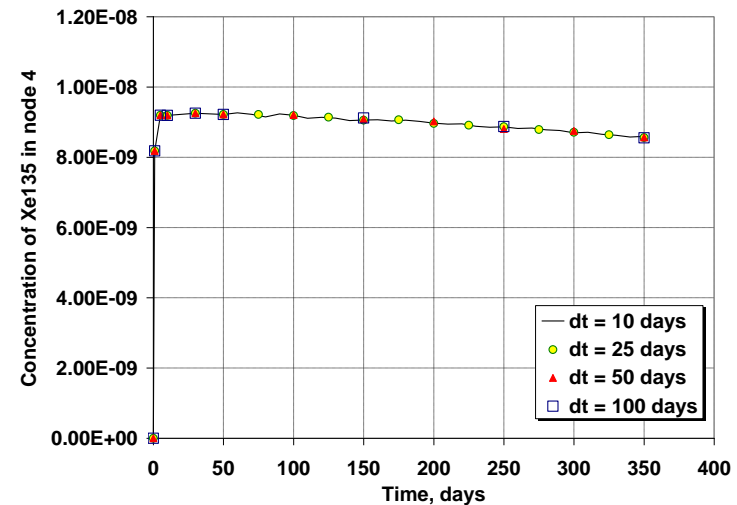
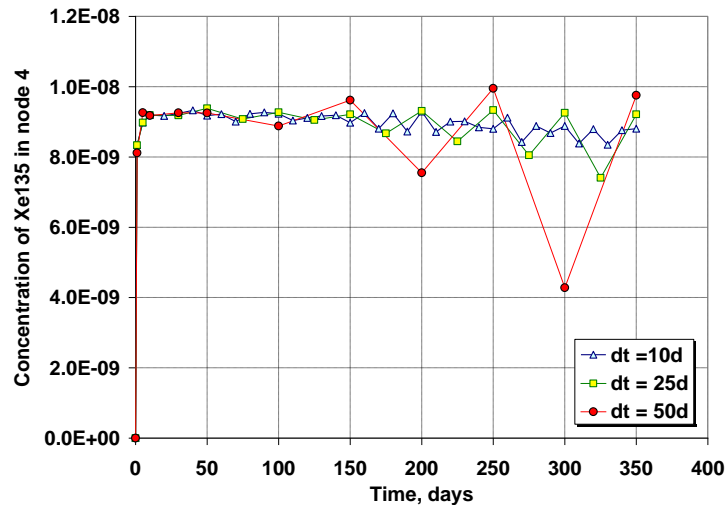
Example of an oscillatory behavior: PWR



□ Spatial oscillatory behavior



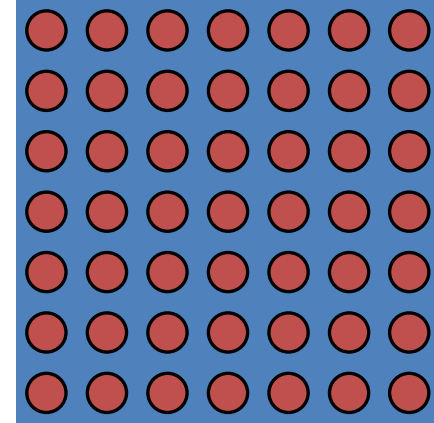
□ BOT vs. SIMP methods



Test Case Description



- ❑ 7×7 BWR assembly, UO_2 fuel
 - 36 axial burnup regions
- ❑ Previous work examined PWR assembly
 - The oscillations developed immediately (BOL)
- ❑ Axial void dist. determines the flux dist. @ BOL
- ❑ Initially, local burnup effects do not affect the flux shape
- ❑ Numerical oscillations can still develop at higher burnup
 - Void dist. effect is compensated by the burnup dist. effect

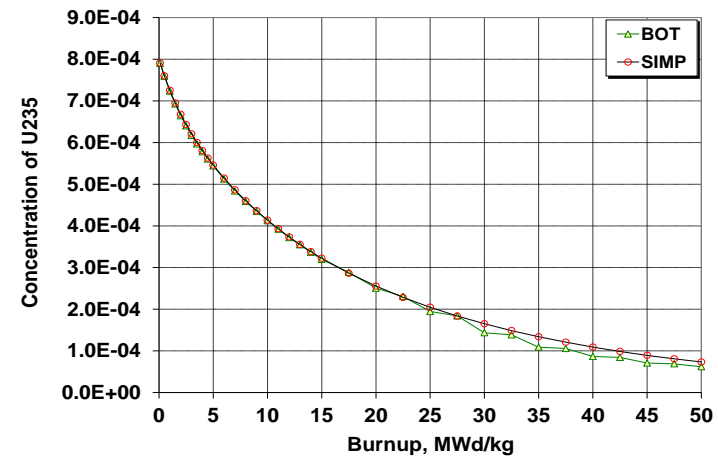
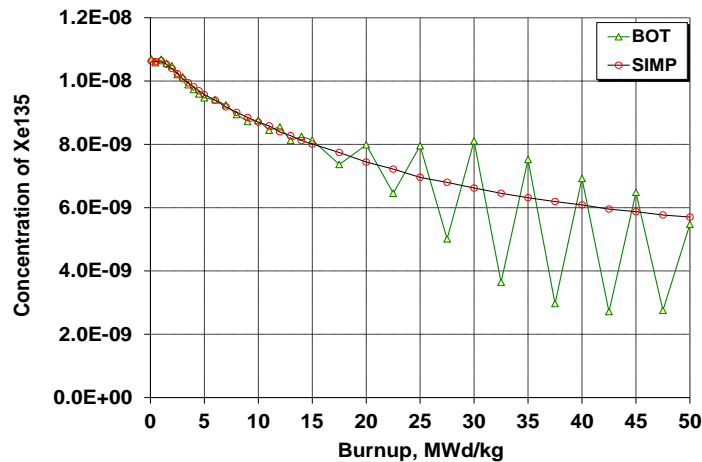
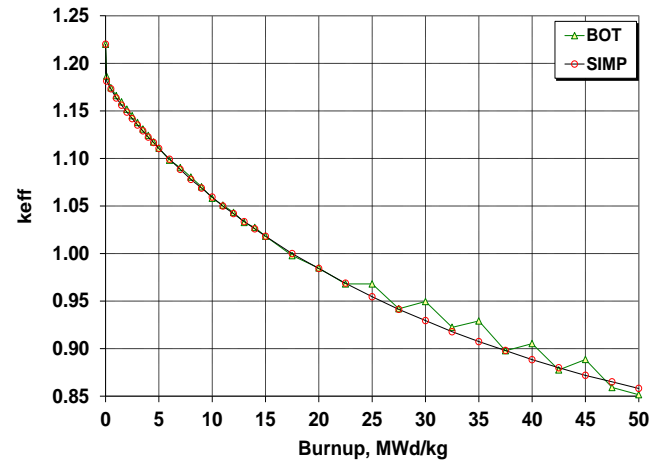


BOT vs. SIMP results



- ❑ BOT coupling scheme becomes unstable after ~ 15 MWd/kg
 - Oscillations in spatial dist. of neutronic and TH parameters
 - Can be visually observed
 - K-eff and nuclide density dist.
 - all oscillate

- ❑ SIMP is numerically stable





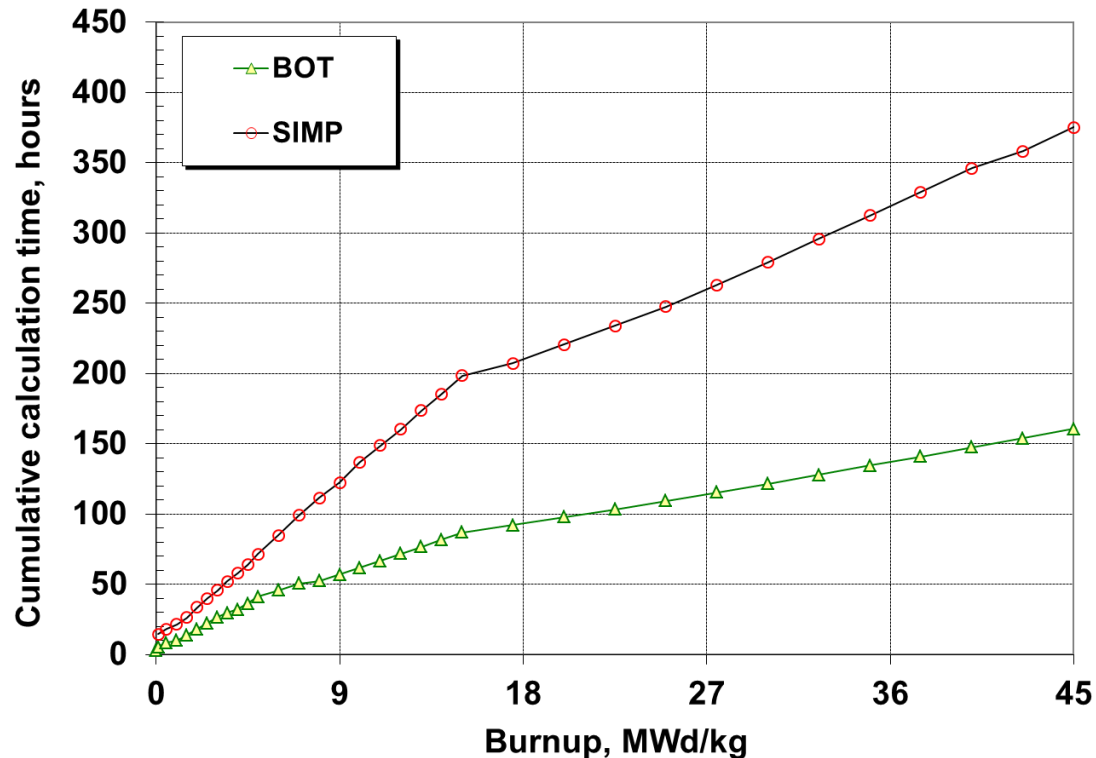
- ❑ Numerical instability of BOT method is a real issue
 - **Was only qualitatively** assessed by visual inspection of the results
 - In some cases (few initial time steps) – no oscillations observed
- ❑ The oscillations disappear if SIMP method is used throughout
 - SIMP is unconditionally stable
 - But, requires more iterations \Rightarrow higher computational cost
 - For cases when only depletion is considered
 - Explicit Euler method \rightarrow 1 MC transport solutions
 - Implicit methods $\rightarrow \geq 3$ MC transport solutions
- ❑ Can the two methods be combined?
 - Use fast BOT but continuously monitor numerical stability
 - Switch to more computationally intensive SIMP if instability is observed

Open issues: computational costs



❑ SIMP requires more iterations than BOT

- Simultaneous convergence on TH & ND
- BOT converges only on TH field



❑ Oscillation problem is case specific

- Oscillation may not appear at all or may develop later
- Therefore, employing implicit methods may be inefficient



- ❑ Diagnostic mechanism is required:
 - To identify the onset of numerical instabilities,
 - To alert the user, **or**
 - Automatically switch to SIMP algorithm
 - BOT (fast and simple) → SIMP unconditionally stable but computationally more expensive
 - Such hybrid algorithm was developed and implemented in BGCore
 - Assures numerical stability
 - Improves computational efficiency of coupled MC codes
 - Does not require any intervention from the user



□ The irradiation time is subdivided into time steps

- At each time-step

- Iteratively solve MC, depletion and TH problems
- The procedure is repeated for the following steps



- The global solution at any base point n is achieved by:
 - Sequentially solving the prior sub-steps ($\leq n$)

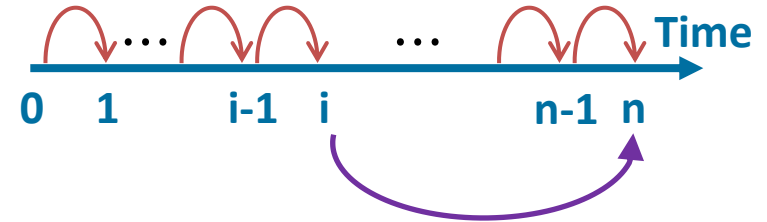
□ Analyzing the behavior of time integration method

- Define amplification (growth) factor: G
- The solution is stable if G is bounded
- Keep monitoring G for the following steps



- ❑ A stable scheme produces a bounded solution if the exact solution is bounded
- ❑ Error between computed solution and the exact solution should not be amplified as we progress in time
- ❑ Notation:
 - \bar{U}^N Exact solution
 - U^N Computed solution
 - ε^N error = $\bar{U}^N - U^N$
- ❑ The stability requires that:
 - $G = \left| \frac{\varepsilon^{N+1}}{\varepsilon^N} \right| \leq 1$
 - The growth factor must satisfy this condition for all time-steps

Computing the stability criterion (1)



□ For a given base point n

- \bar{U}^n (RR dist.) is approximated by the finest available time-steps set

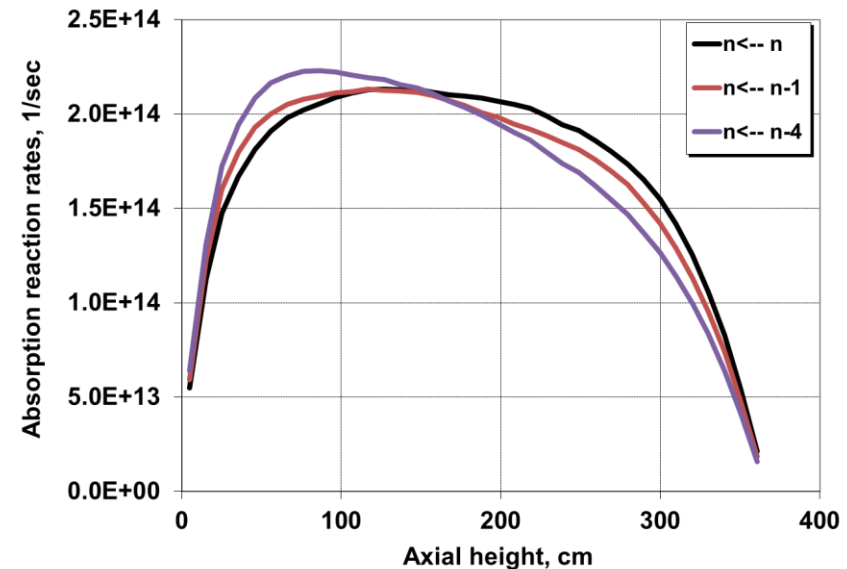
- $U^{n \leftarrow i}$ RR at point n are obtained by re-depleting:

- Starting from point i , with N^i and ϕ^i and $\Delta t = t^n - t^i$

□ Calculate the error in RR

- $\delta^n = |\bar{U}^n - U^{n \leftarrow n-1}|$

- $\delta^i = |\bar{U}^n - U^{n \leftarrow i}|$



Computing the stability criterion (2)



□ Calculate G_i

- $G_i = \frac{\delta^n}{\delta^i}$

- Repeat the procedure of calculating G_i for all $i \in [0, n - 2]$

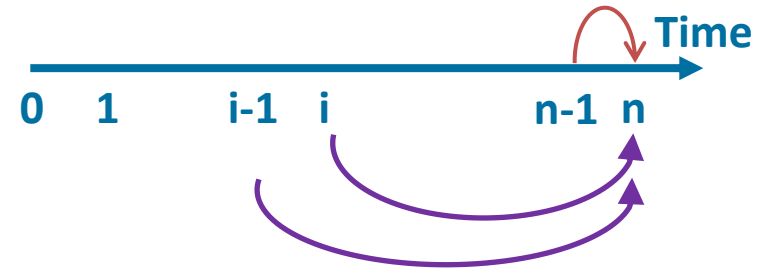
□ The quantity of interest is: $G = \max\{G_i\}$

- The error should not be amplified regardless of the step-size (i)

- i.e. **all the solutions for different Δt must be bounded**

□ The scheme is stable if $G \leq 1$ and unstable otherwise

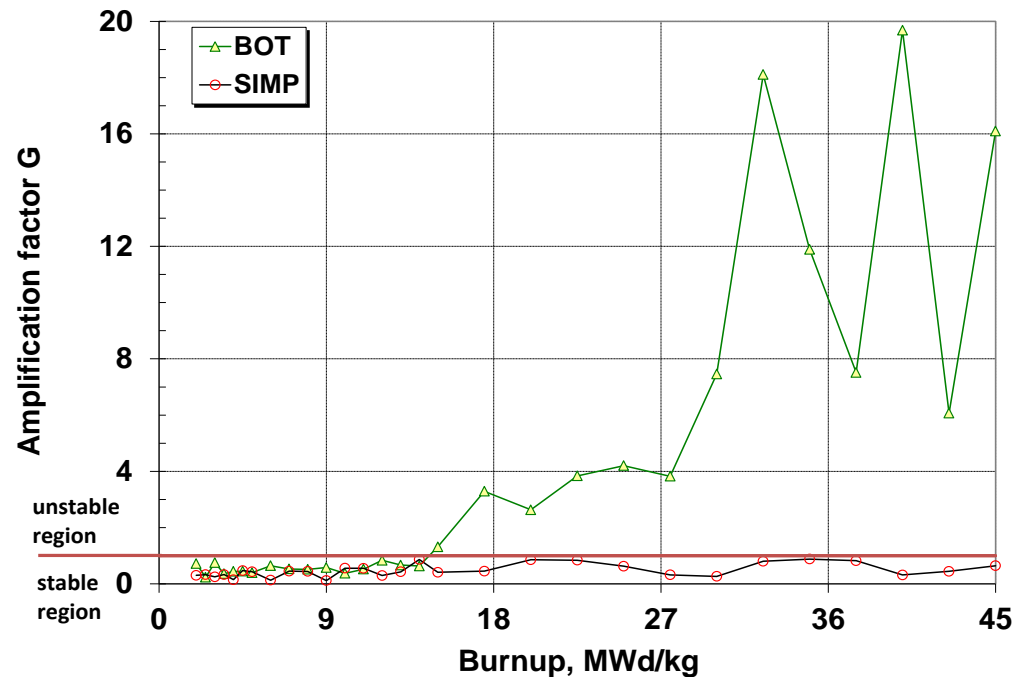
□ The procedure is repeated for each time point n



Results: amplification factor G



- Quantitative assessment of the stability
 - SIMP is stable ($G < 1$)
 - BOT is stable only up until ~ 15 MWd/kg



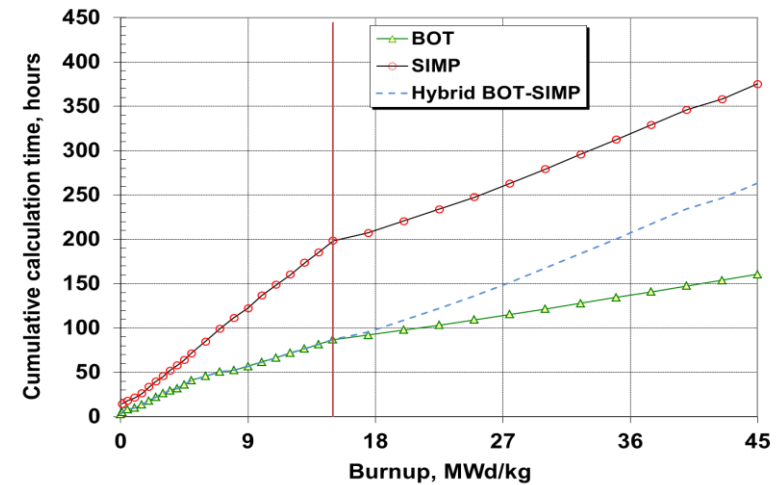


□ Total (cumulative) execution time until the onset of oscillations:

- **BOT** 87 hr. (135 transport solutions)
- **SIMP** 198 hr. (278 transport solutions)
- Applying BOT (<15 MWd/kg) and SIMP thereafter saves:
 - 111 hr. (143 transport solutions)

□ CPU costs for calculating G

- Loading XS data
- Solving Bateman equations
- Matrix-Vector multiplication



Conclusions (1)



- ❑ Existing MC coupling methods may be unstable
- ❑ Stochastic implicit mid-point (SIMP) methods was developed
 - Unconditionally stable
 - But, more computationally intensive
- ❑ Some problems do not have stability issues
 - Always using SIMP would be a waste of computing resources

Conclusions (2)



- ❑ A method for monitoring numerical stability was developed
 - Evaluates error amplification factor which must be bounded
 - Capable of identifying the onset of instability
 - Can automatically trigger the transition:
 - From: Explicit BOT
 - To: Implicit SIMP method
- ❑ The hybrid method is more computationally efficient
- ❑ Computational requirements for monitoring stability are negligible



Thank you for your attention

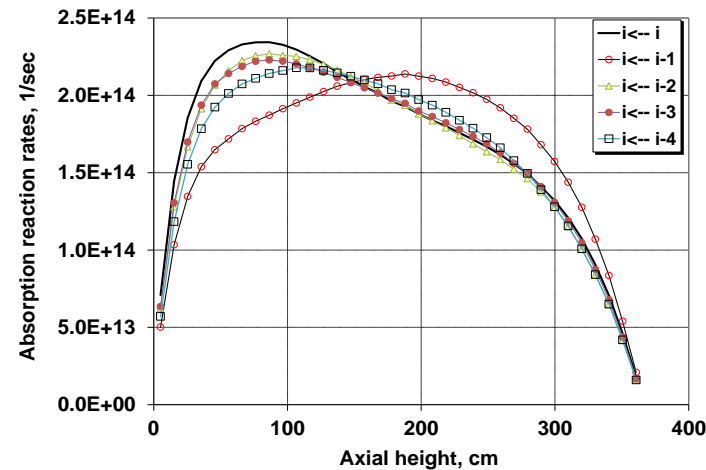
Absorption reaction rate distribution



Comparison of BOT and SIMP methods

Absorption reaction rate

BOT method



SIMP method

