

Variance reduction for collimated gamma detector geometry

14/09/2022 VTT – beyond the obvious

Presenting a new variance reduction method

- What was it developed for?
 - To increase the performance of passive gamma emission tomography simulations
- Based on physics special assignment
 - Report available in Aalto University archive:
<http://urn.fi/URN:NBN:fi:aalto-202206234186>

Outline

- Background and motivation
- Altering the photon tracking routine
- Demonstration

Background and motivation

PGET

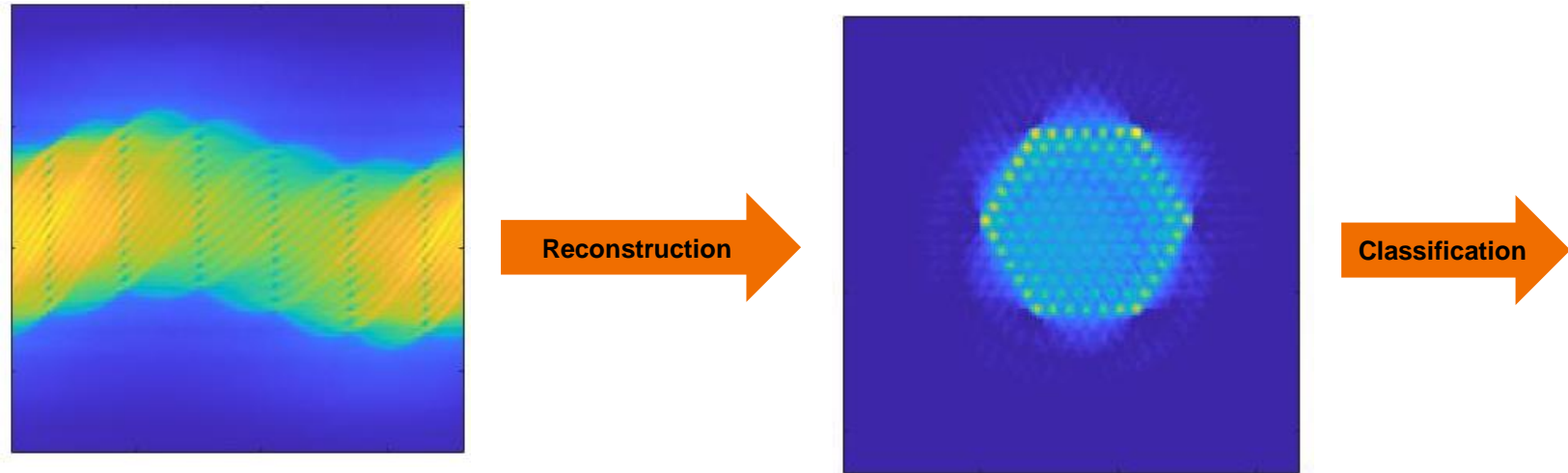
Passive
Gamma
Emission
Tomography

- Safeguards
- Relative to medical imaging SPECT
- Submerged measurements
- Final disposal



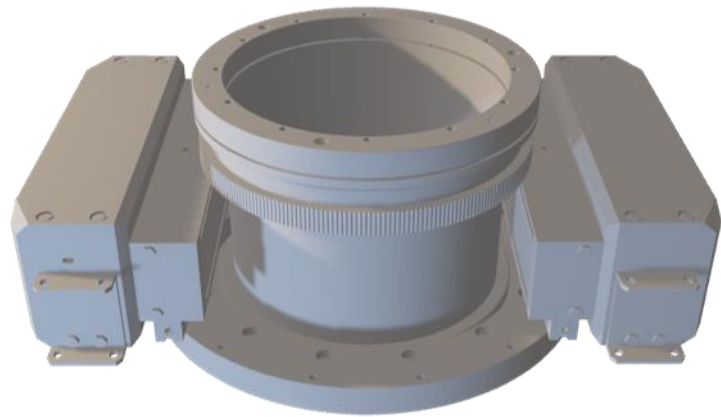
Image: Dean Calma / IAEA <https://flic.kr/p/2iFpck3>

Classification process in a nutshell



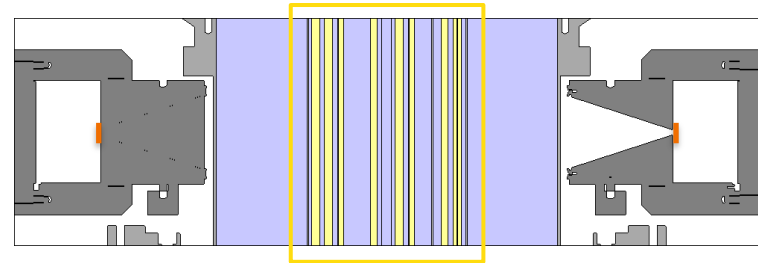
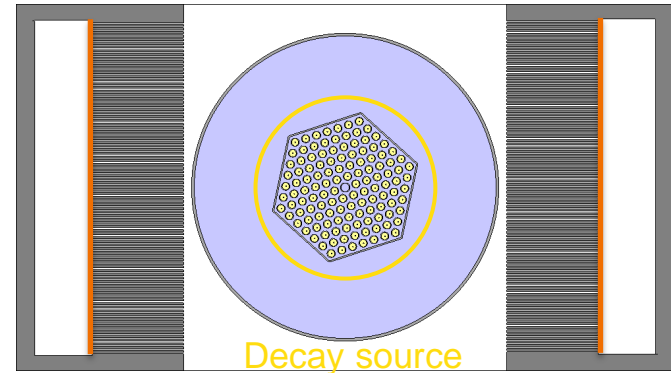
Images: Riina Virta, Finnish Radiation and Nuclear Safety Authority STUK

Serpent model is based on hybrid CSG and CAD geometry



CAD model from STUK

Photon current detectors



Analog photon transport in the geometry is inefficient

- Only a fraction of all simulated histories pass the collimator
- Fuel itself is highly attenuating
- Transport is repeated several times
 - Fuel assembly is rotated
 - 3 degree spacing sufficient
 - 120 independent calculations
 - One calculation takes from days to weeks on one node

Analog simulation possible but impractical

Altering the tracking routine

Variance reduction on abstract level

- Variance of an implicit MC estimate $s_{h'}^2$ can be related to the analog estimate s_h^2 approximately as¹

$$s_{h'}^2 \approx QF^{-1} \frac{D}{D'} s_h^2$$

Expected number of scores with analog sampling

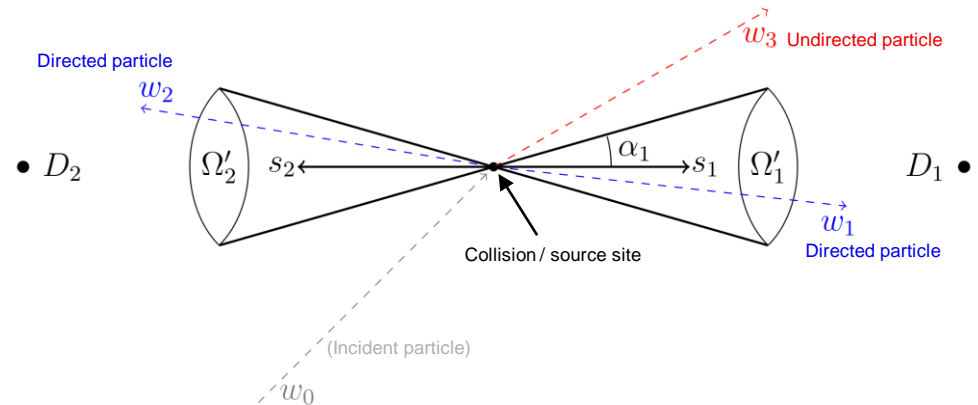
Quality factor ≤ 1

Expected number of scores with non-analog sampling

To reduce variance, increase the number of scored histories while avoiding a high spread of particle weights (bad QF)

Directional splitting was developed to increase probability of passing collimator

- Influenced by forced detection ^{1,2}
- Applied for source sampling and Compton scattering
- Directed particles are sampled uniformly within cones
- Undirected particle keeps the simulation consistent



¹D. R. Haynor, R. L. Harrison, and T. K. Lewellen, "The use of importance sampling techniques to improve the efficiency of photon tracking in emission tomography simulations," *Medical Physics*, vol. 18, no. 5, pp. 990–1001, 1991

²J. De Beenhouwer, S. Staelens, S. Vandenberghe, and I. Lemahieu, "Acceleration of gate spect simulations," *Medical Physics*, vol. 35, no. 4, pp. 1476–1485, 2008.

Statistical weights have to be adjusted for each fragment

- Total weight is conserved $w_0 = w_1 + w_2 + w_3$
- Proportional to the probability of sampling the particle to the given direction
- Isotropic decay source $w_{dec} = \frac{\Omega}{4\pi}$
- Compton scattering

$$w_{co} = \frac{1}{\sigma_{co}} \int_{\Omega'} \left(\frac{d\sigma_{co}}{d\Omega} \right)_{KN} S(q, Z) d\Omega$$

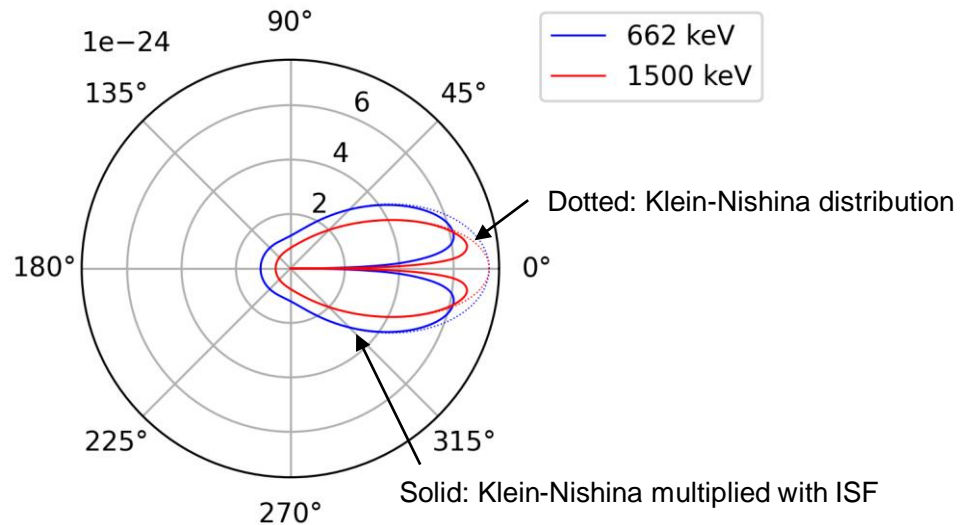
Compton cross section

Double differential cross section (Klein-Nishina)

Incoherent scattering function

Serpent utilizes modified Klein-Nishina distribution for Compton scattering¹

- Example of scattering with uranium
- Incoherent scattering function suppresses forward scattering
- Weight calculation requires numerical integration



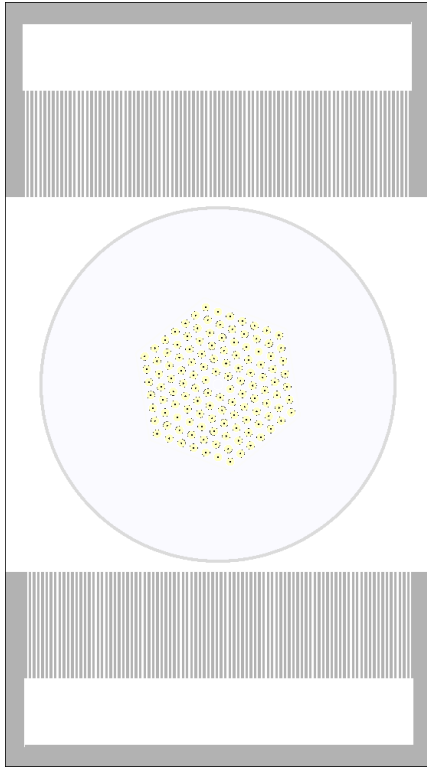
¹ T. Kaltiainenaho, "Photon transport physics in Serpent 2 Monte Carlo code," Computer Physics Communications, vol. 252, pp. 107–143, 2020

Details of the implementation

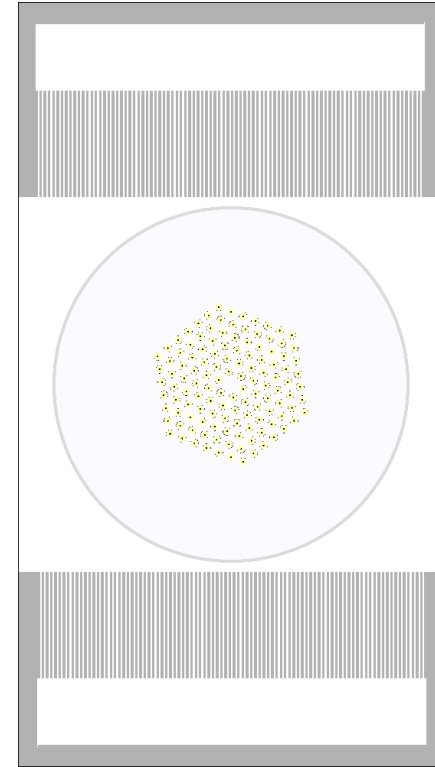
- Branching is controlled with lower energy limit and Russian roulette
- Premature stage: lot of hard-coded parameters
 - Compton scattering weights tables
 - Size of bounding cones for directed particles
 - Region where the method is applied
 - Weight limit for the Russian roulette
- Secondary particles and their effects in Compton scattering cannot be reliably taken into account
- Currently only the author has the code

Demonstration

Animation: 1000 initial photons, XY cross section



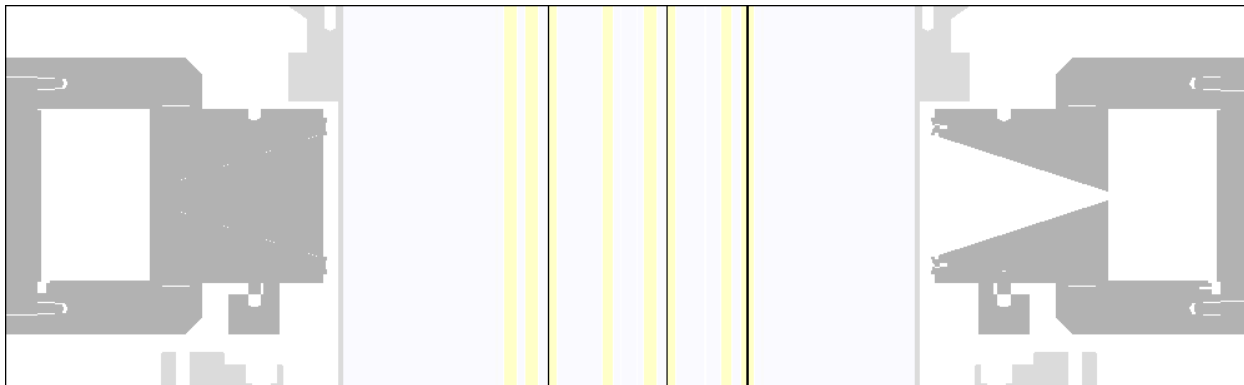
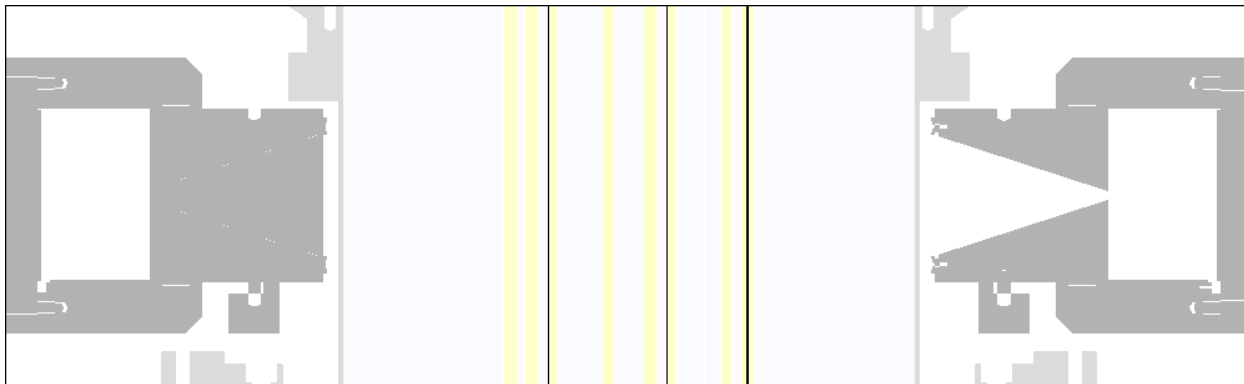
Analog



Directional splitting

Animation: 1000 initial photons, YZ cross section

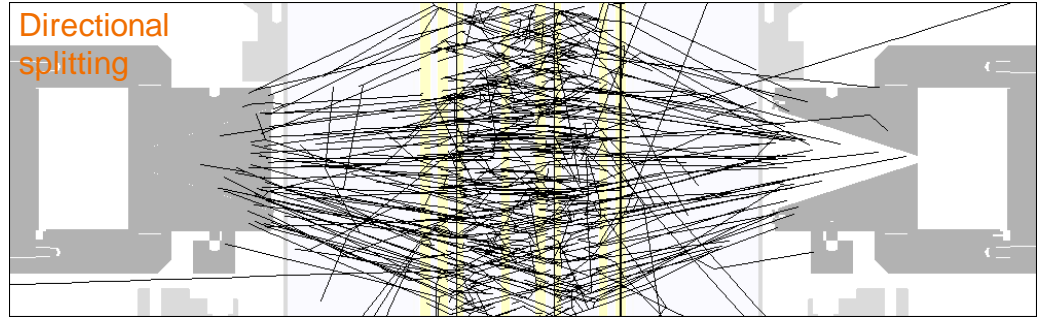
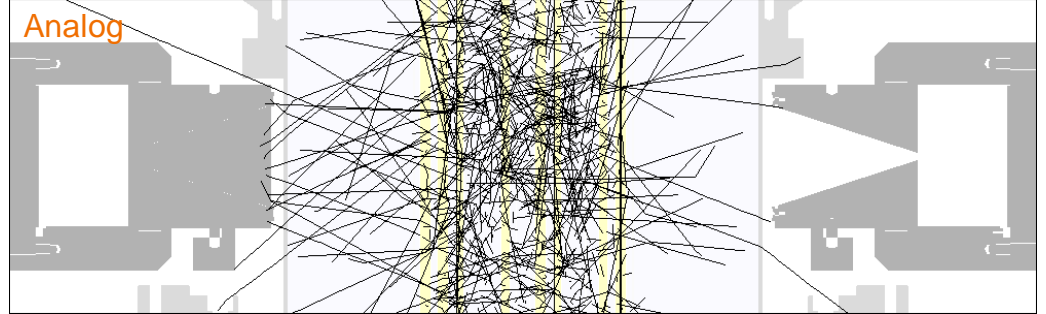
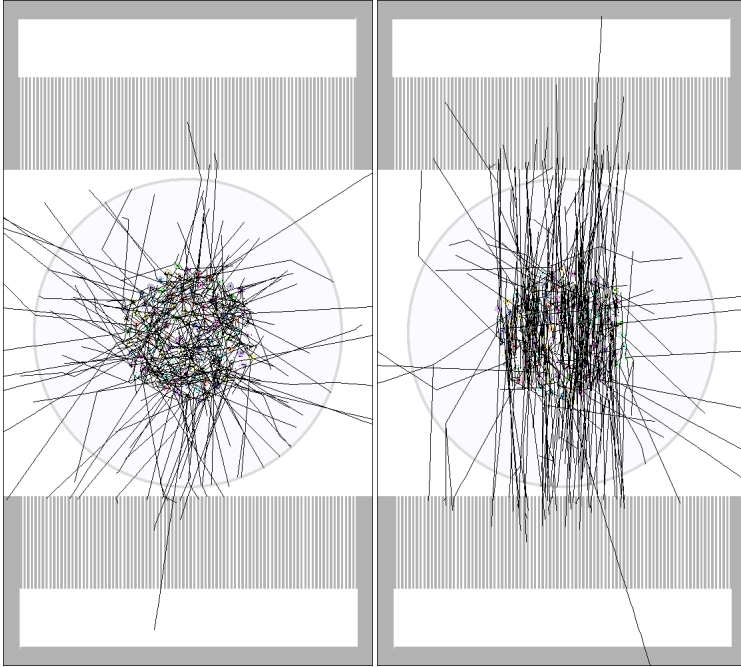
Analog

Directional
splitting

(Backup plan: track plots)

Analog

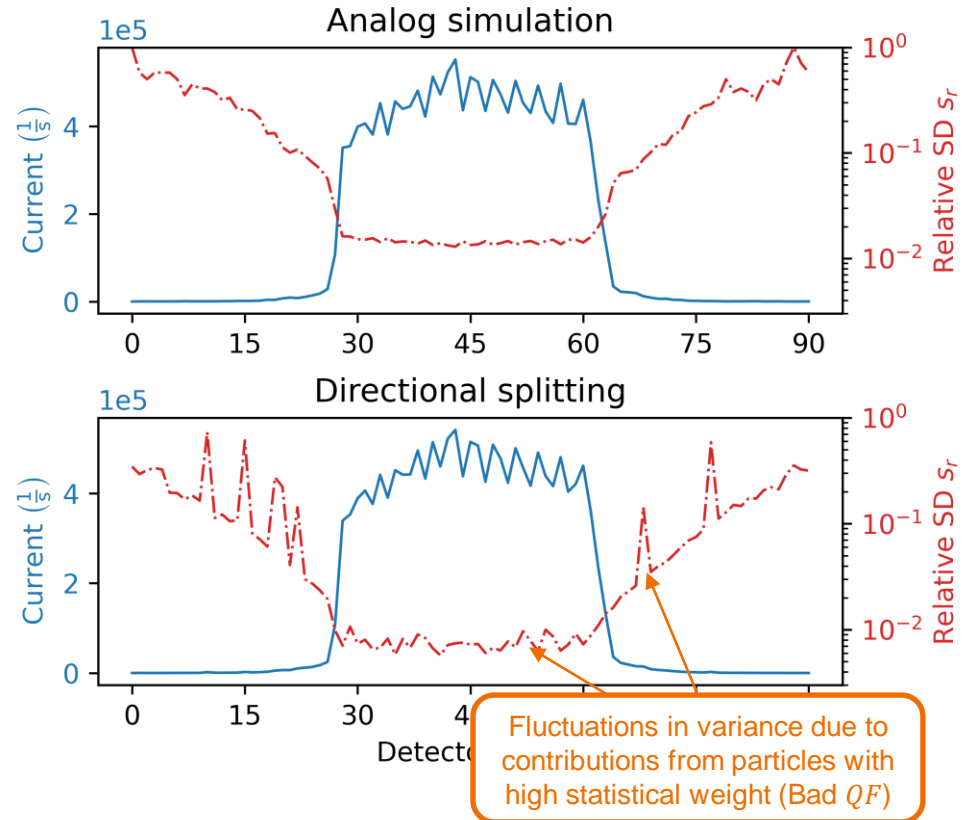
Directional splitting



The method is not perfect but it works

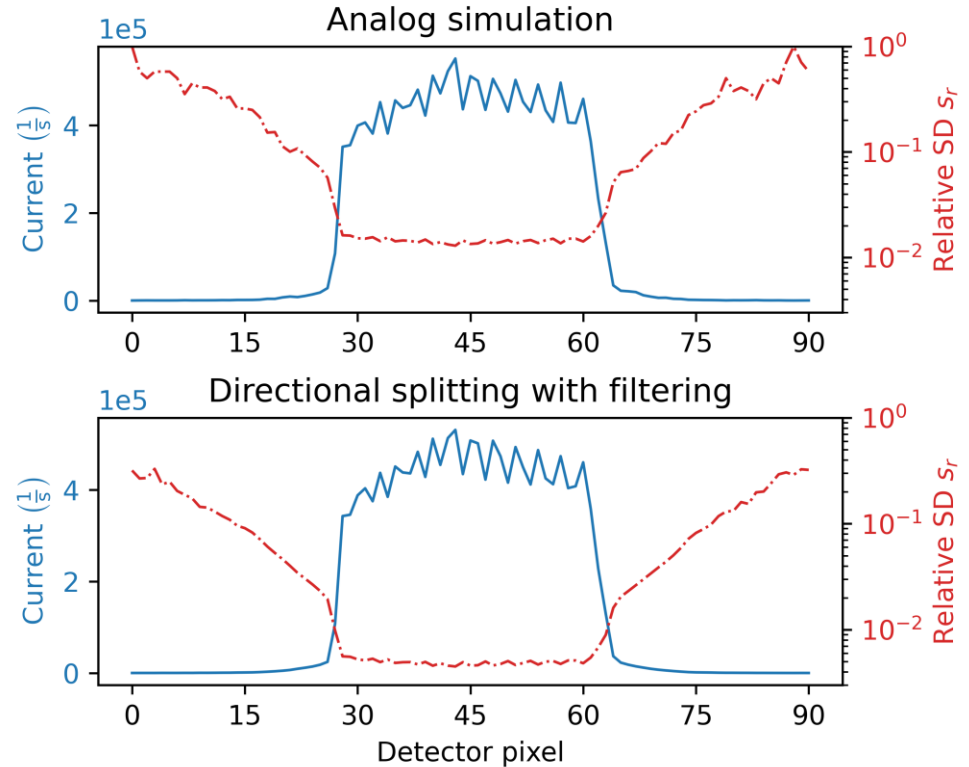
- Range 600 – 700 keV
- Figure of merit $FOM = \frac{1}{s^2 T}$
 - Analog = 0.58
 - Directional splitting = 7.5

- Efficiency improvement
 - Directional splitting = 12.9



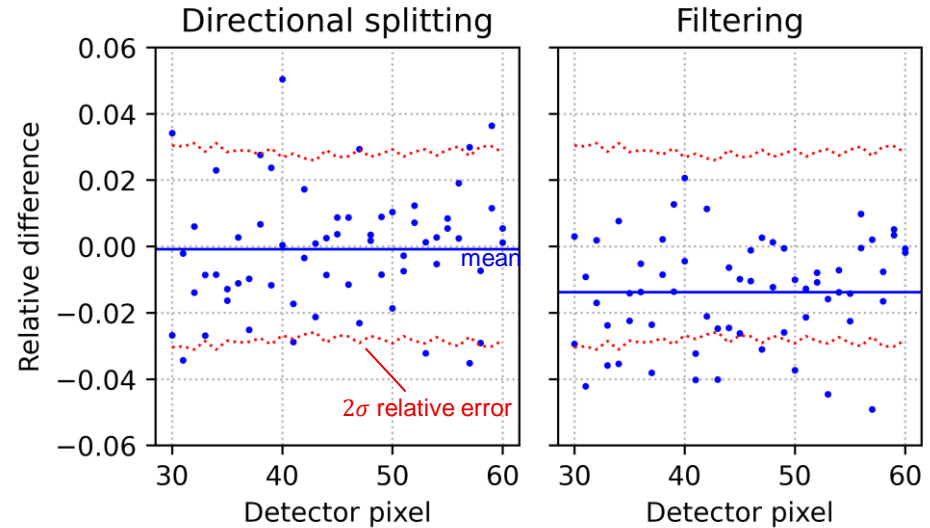
The method is not perfect but it works

- Range 600 – 700 keV
- Figure of merit $FOM = \frac{1}{s^2T}$
 - Analog = 0.58
 - Directional splitting = 7.5
 - ... with filtering = 16
- Efficiency improvement
 - Directional splitting = 12.9
 - ... with filtering = 27.6



Directinal splitting agrees with analog simulation

- Well within statistical uncertainty of analog simulation
- Filtering heavy particle contributions is biased
 - In this case results are underestimated 1 - 2 %



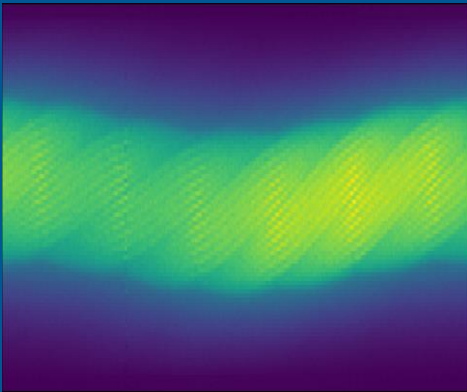
Summary

Summary

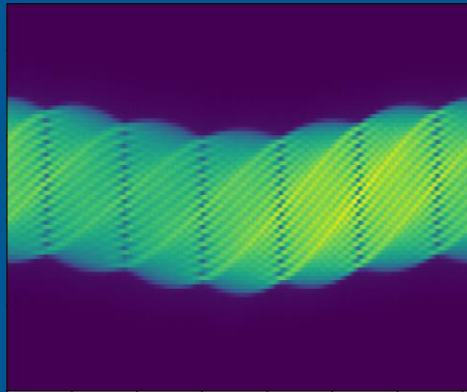
- Directional splitting improved the efficiency of PGET simulations remarkably
- Success of variance reduction is sensitive to weight fluctuations
- Discarding “heavy” particle contributions was a successful strategy in this application, but should be used with caution
- Stage of development premature

Thank you!

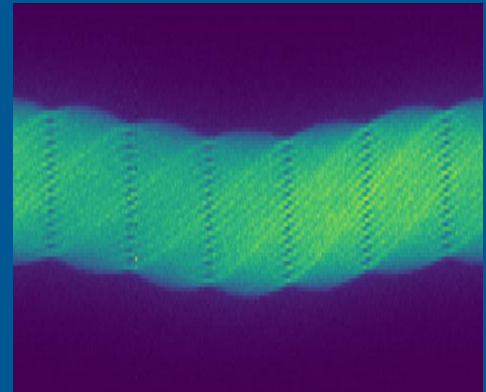
400 – 600 keV



600 – 700 keV



700 – 1500 keV



bey⁰nd

the obvious

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